Measurements in the phase space

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ILLUMINATIO SOLUTIONS

OPTICAL DESIGN

JMO

Illumination Optics

Outline

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OPTICAL DESIGN

- The phase space of illumination optics
  - Phase space diagrams
- Etendue
  - Calculus
  - Fuzzy limits
  - Etendue measurement
- Filling the phase space with luminance
  - Projections
  - Phase space scanning
- Fuzzy source, fuzzy target ?

The phase space of Illumination Optics

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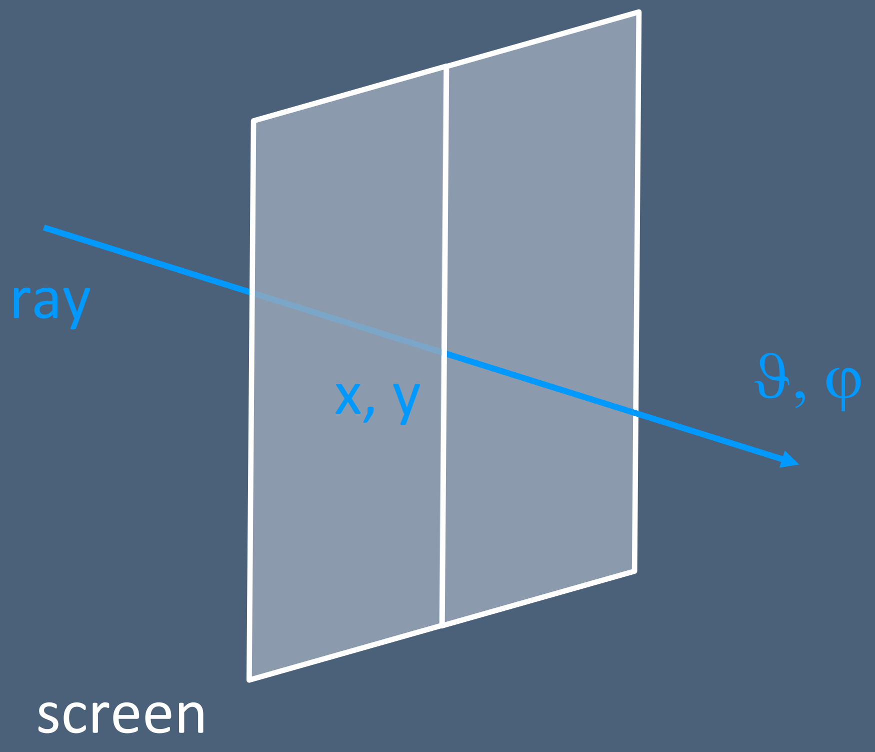

Ray ∈ 4D

x, y, θ, φ

Phase Space

2D cross sections of the 4D phase space

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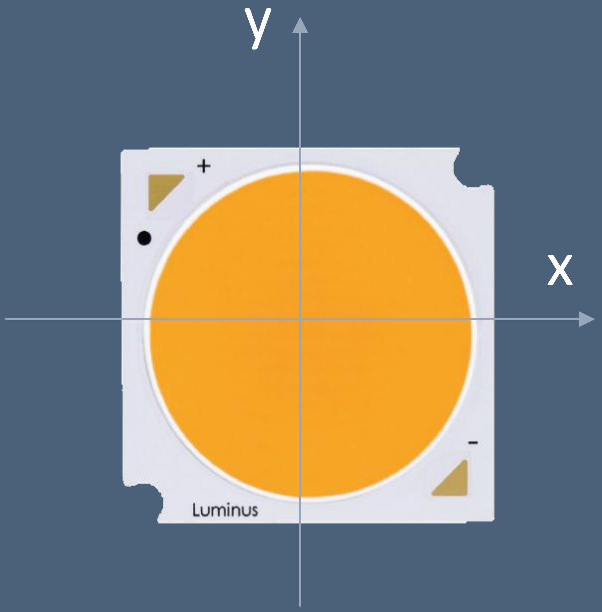
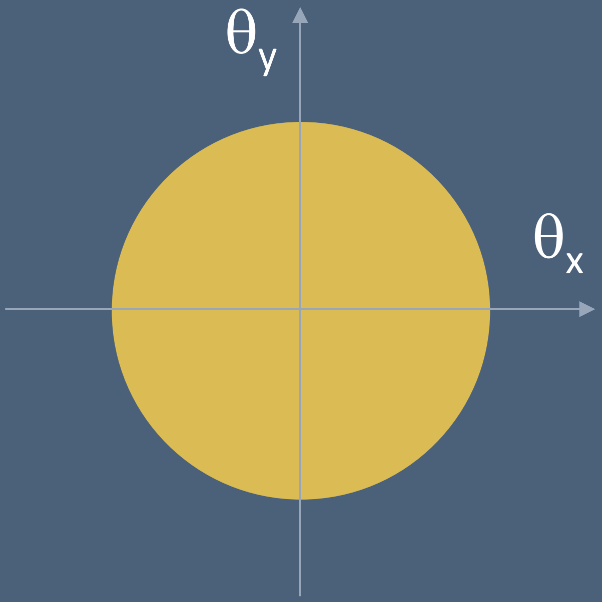
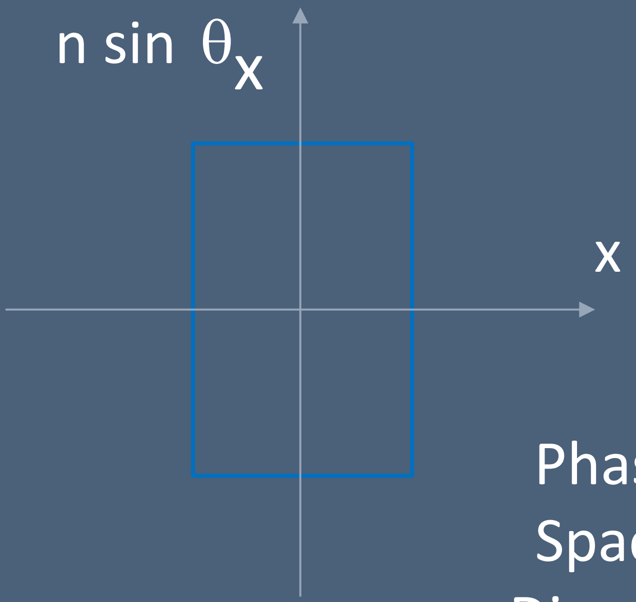






Obvious but less interesting

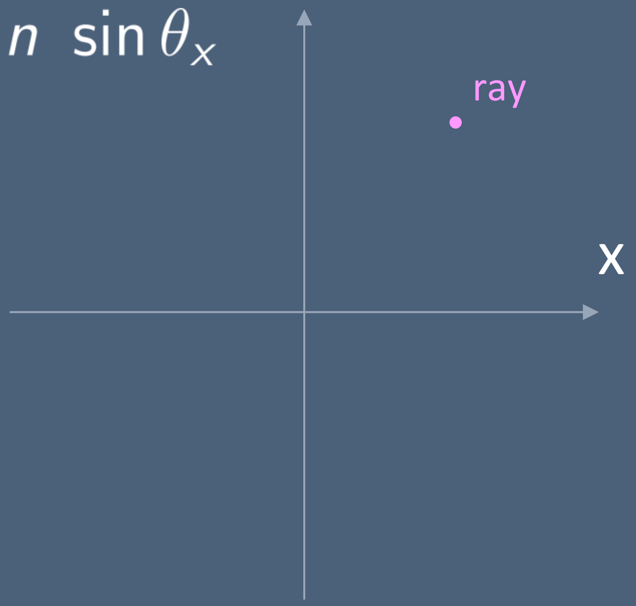
Phase Space Diagram

PSD

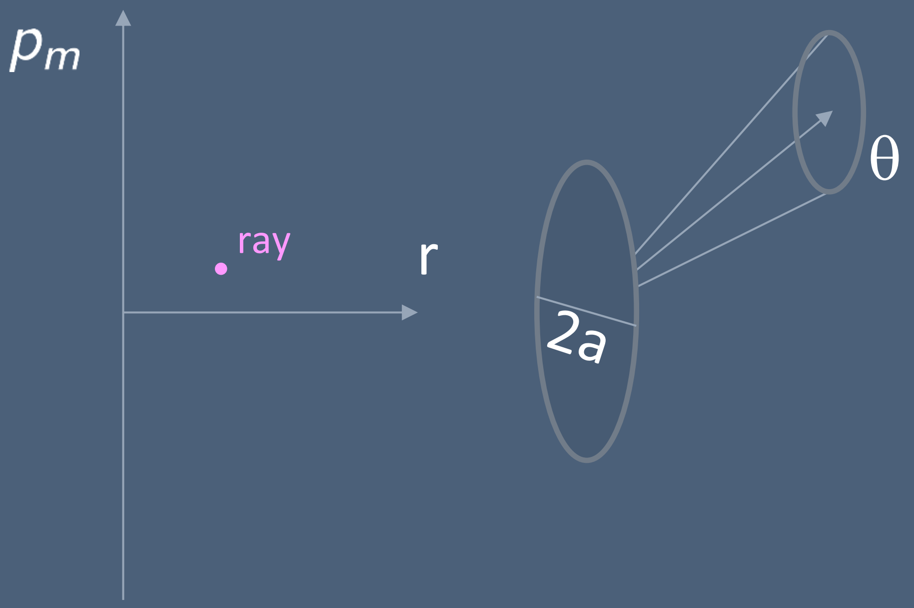
Phase space diagrams

Meridional section PSD

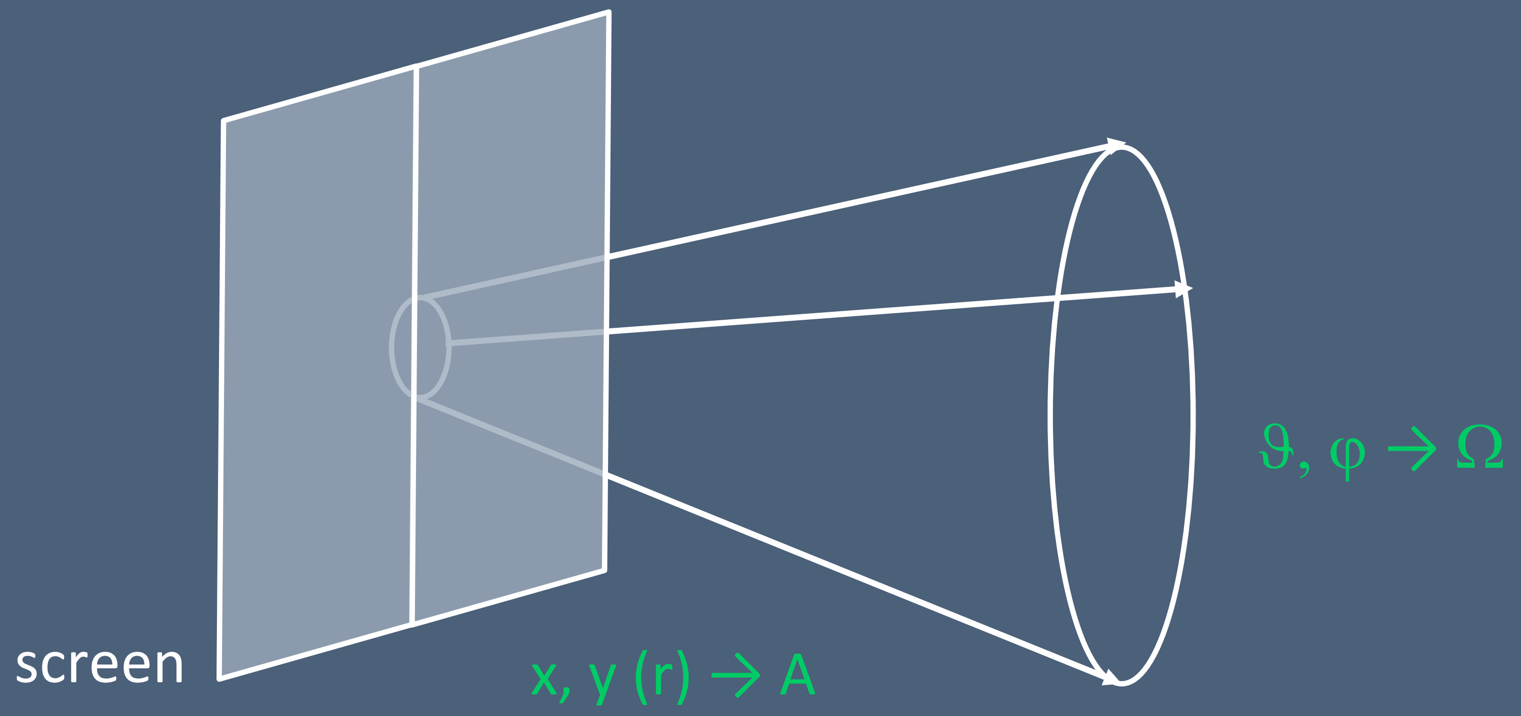
$$p_x = n \ \sin\theta_x$$



Rotationally symmetric PSD



Etendue as a volume in the phase space



$$U = \iint n^2 \cos\theta \; d\Omega dA.$$

Etendue as a volume in the phase space

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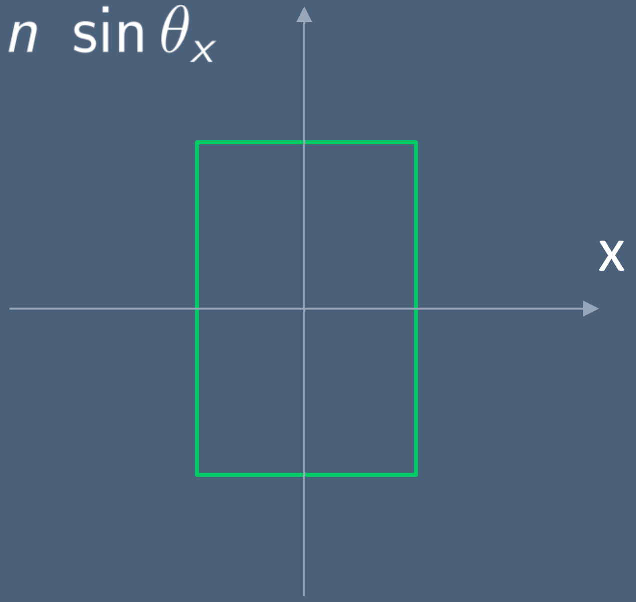
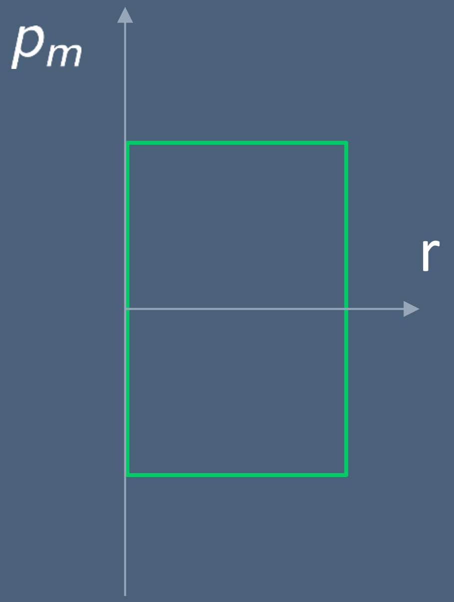
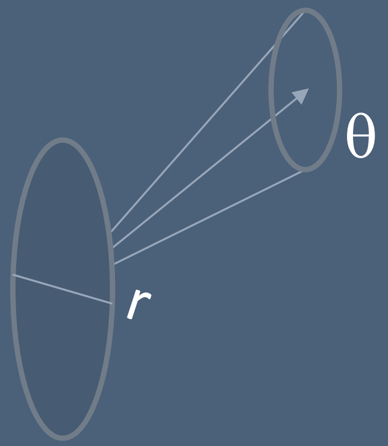
Meridional section PSD

$p_x = n \; \sin\theta_x$



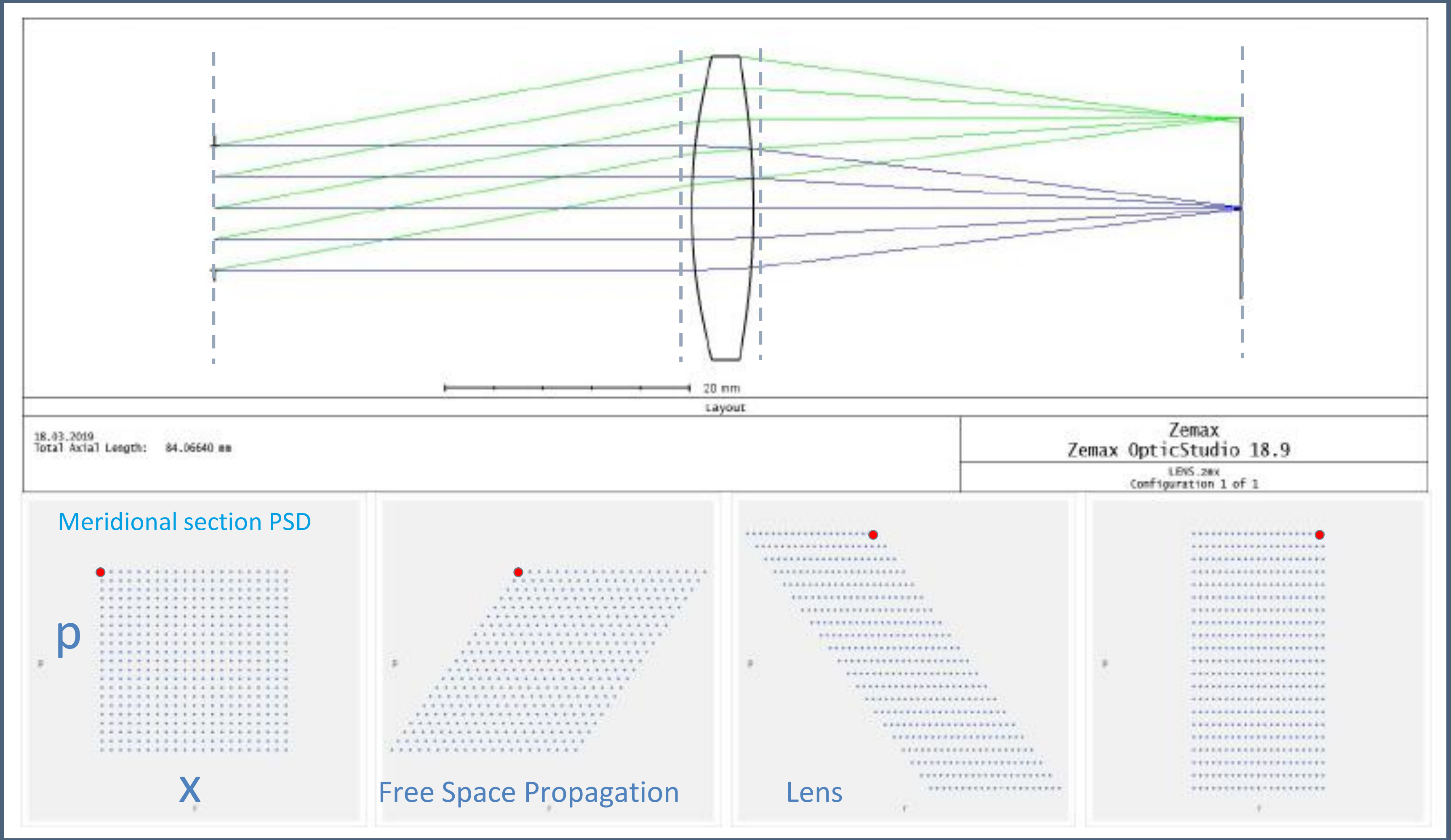
Rotationally symmetric PSD



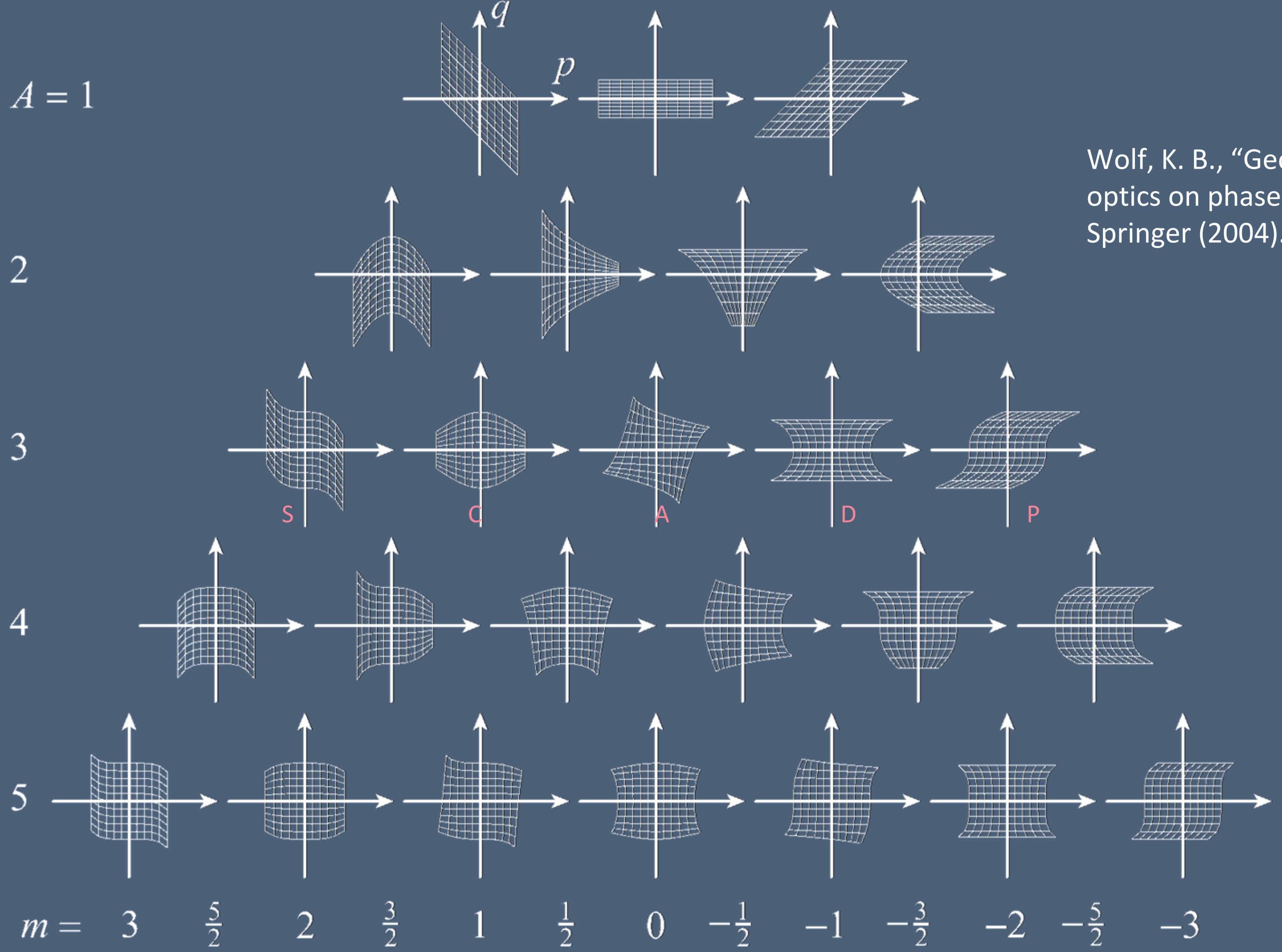


Still geometry only, may be filled with light or not

Optics as a phase space transformer



All phase space transformations



Wolf, K. B., "Geometric optics on phase space", Springer (2004).

Filling the etendue with light

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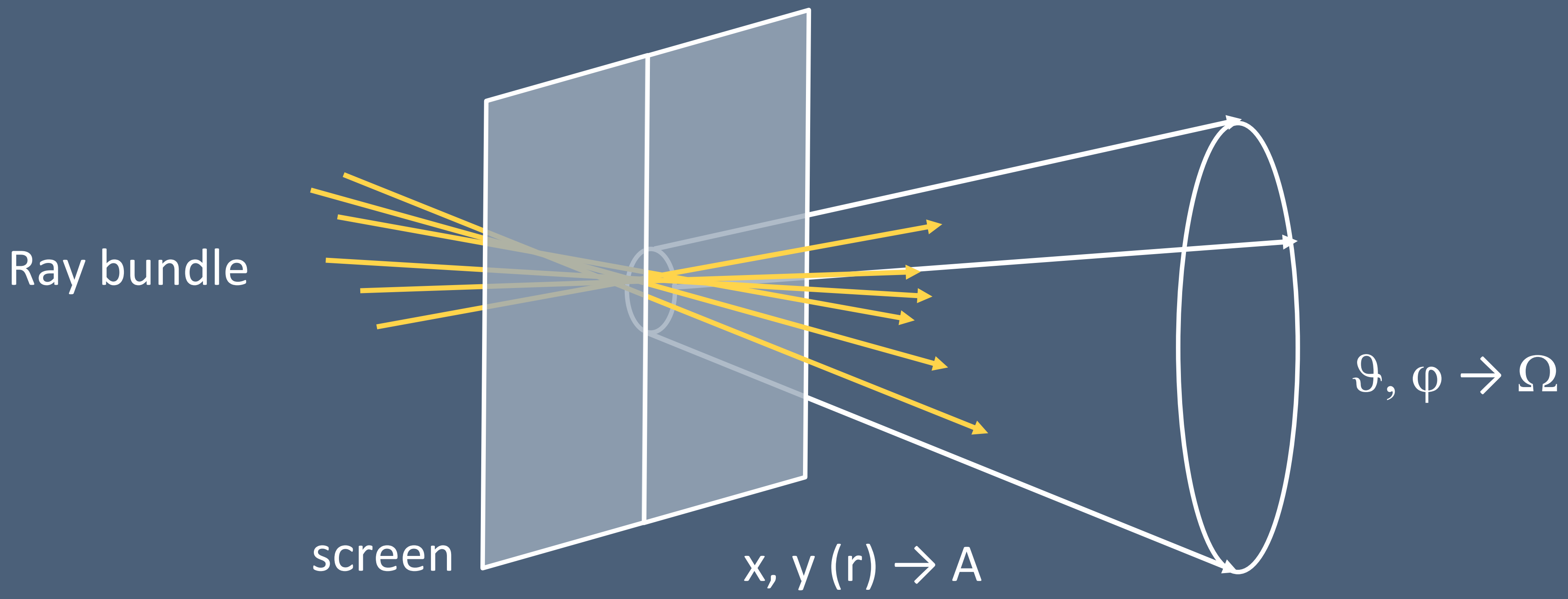

Filling with constant luminance

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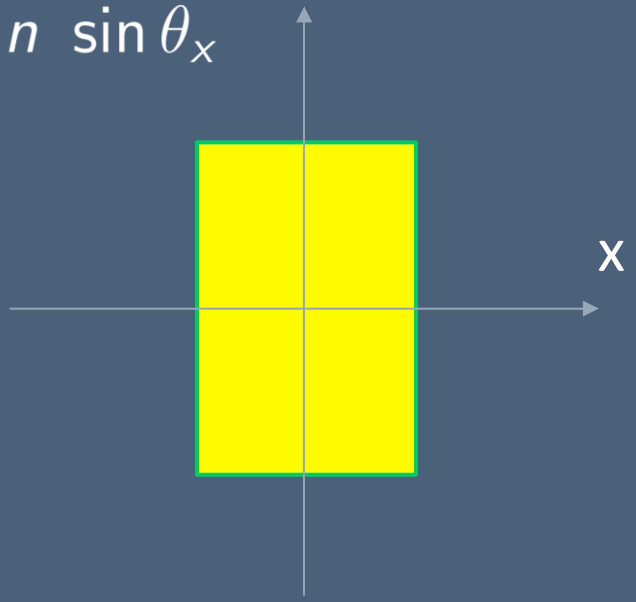
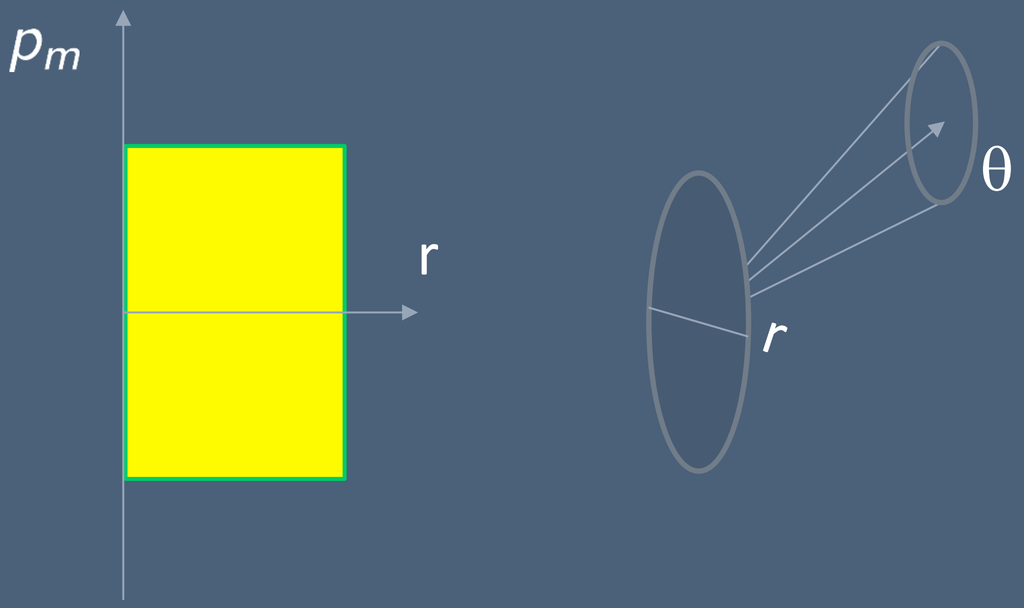
Meridional section PSD

$$p_x = n \sin\theta_x$$



Rotationally symmetric PSD



Filling with variable luminance

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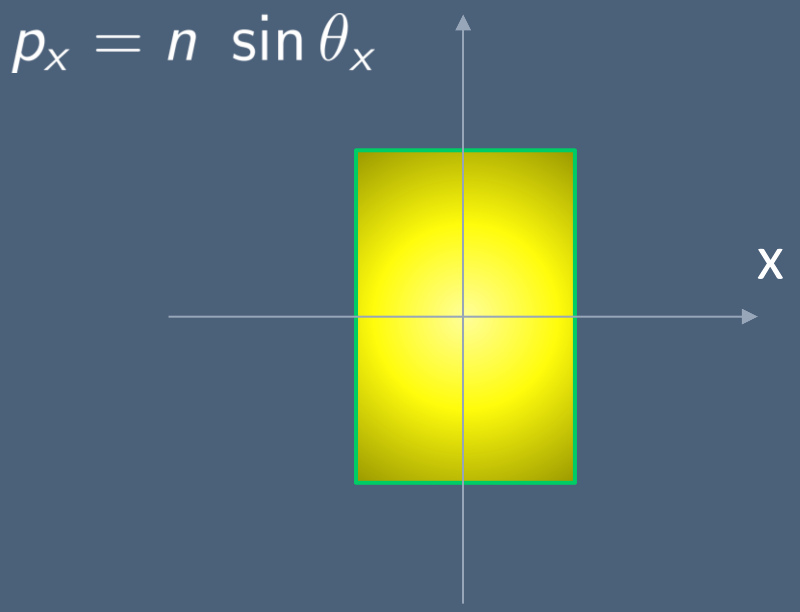
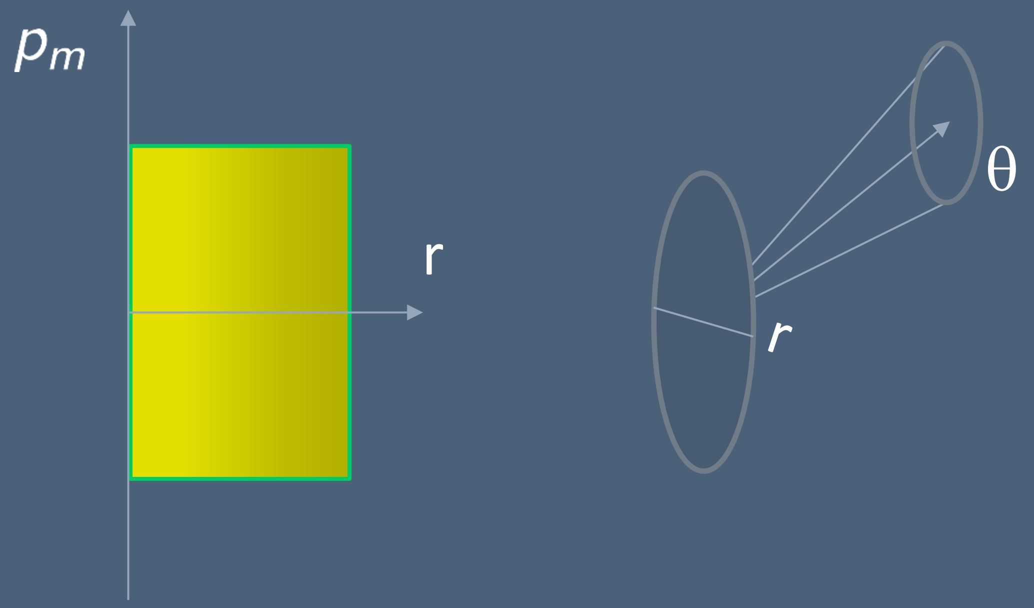
Meridional section PSD

Rotationally symmetric PSD





Photometry as projection from the phase space

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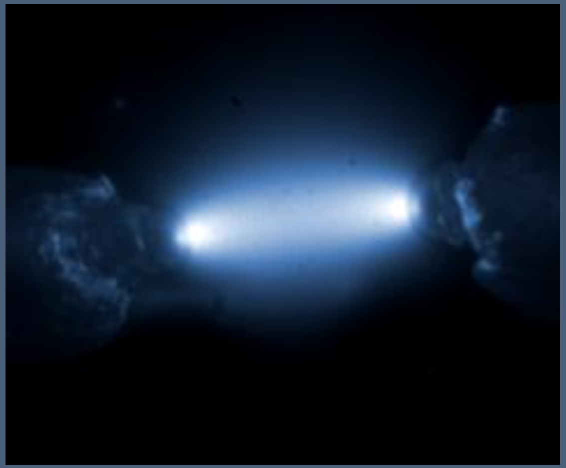

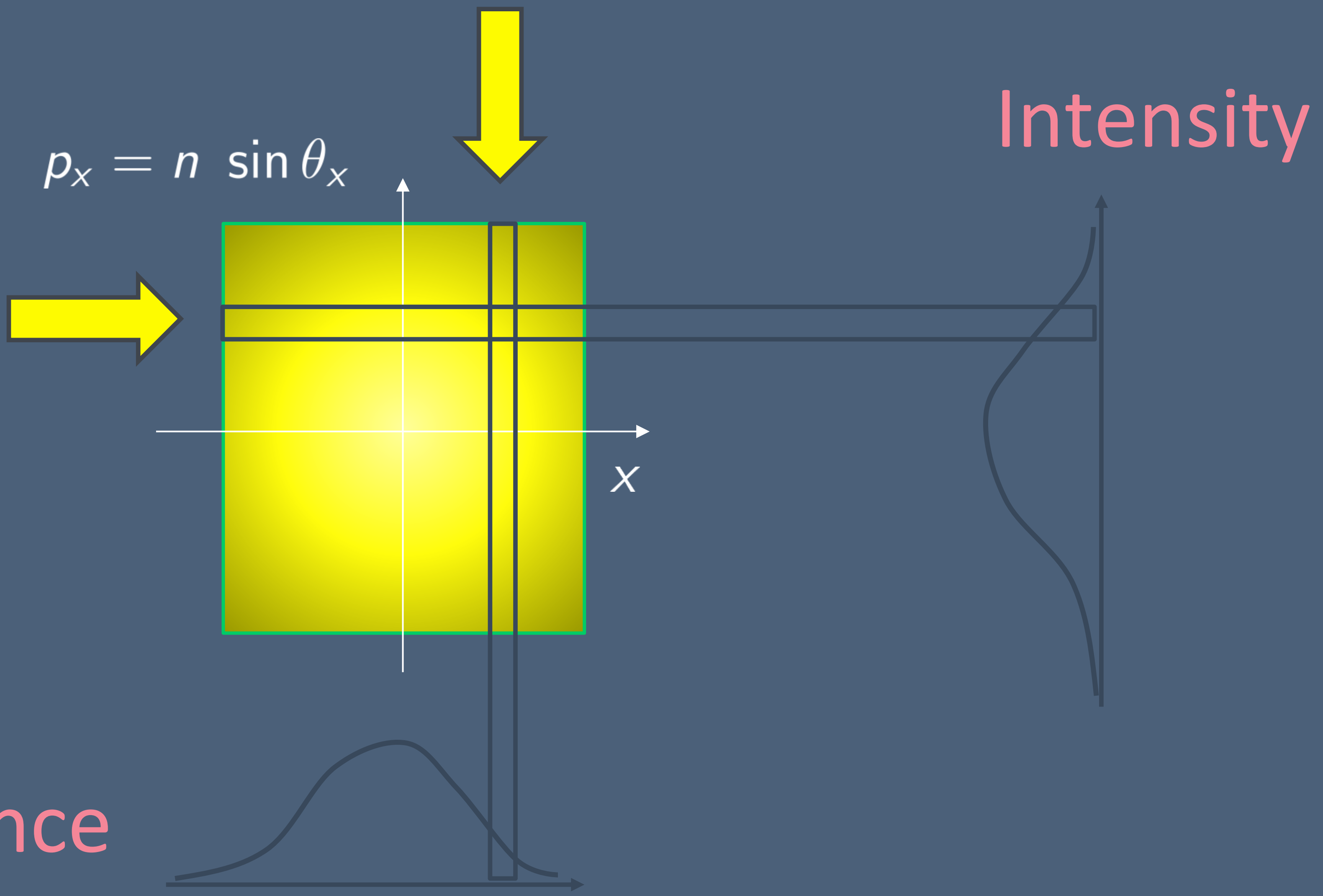

Illuminance

Hard etendue limits: analytic formulas

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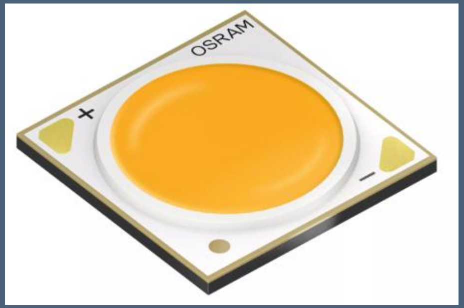
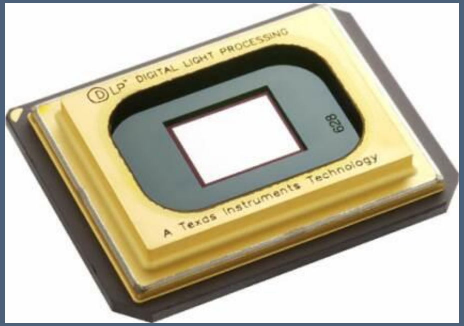
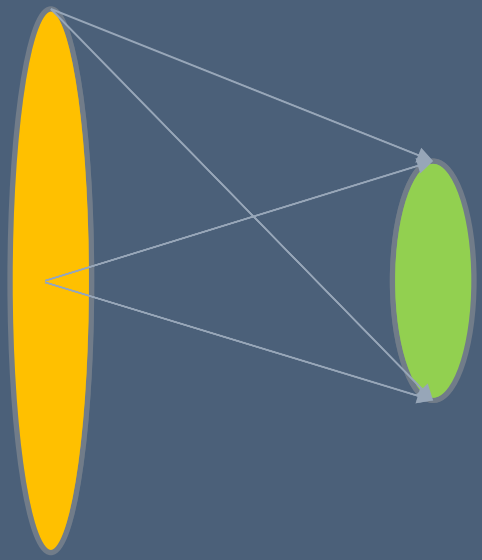
$$U = \iint n^2 \cos\theta \; d\Omega dA.$$



$$U = (n\,\pi\,r\,\sin\theta)^2\; sr$$



$$U = (n \sin\theta)^2 \pi sr \cdot A$$



$$U = \frac{\pi^2}{4}\left(\sqrt{(r+R)^2+D^2} - \sqrt{(r-R)^2+D^2}\right)^2 sr$$

Irregular cases

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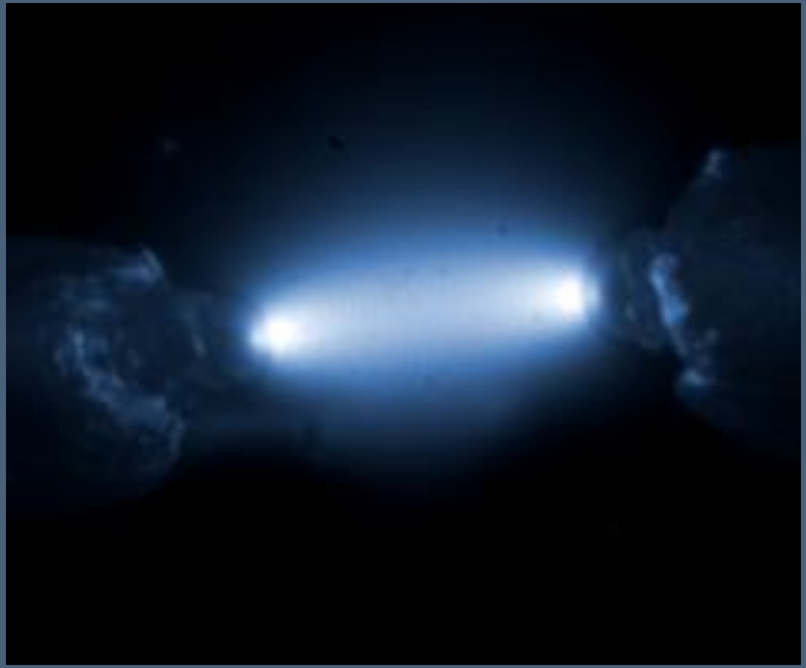
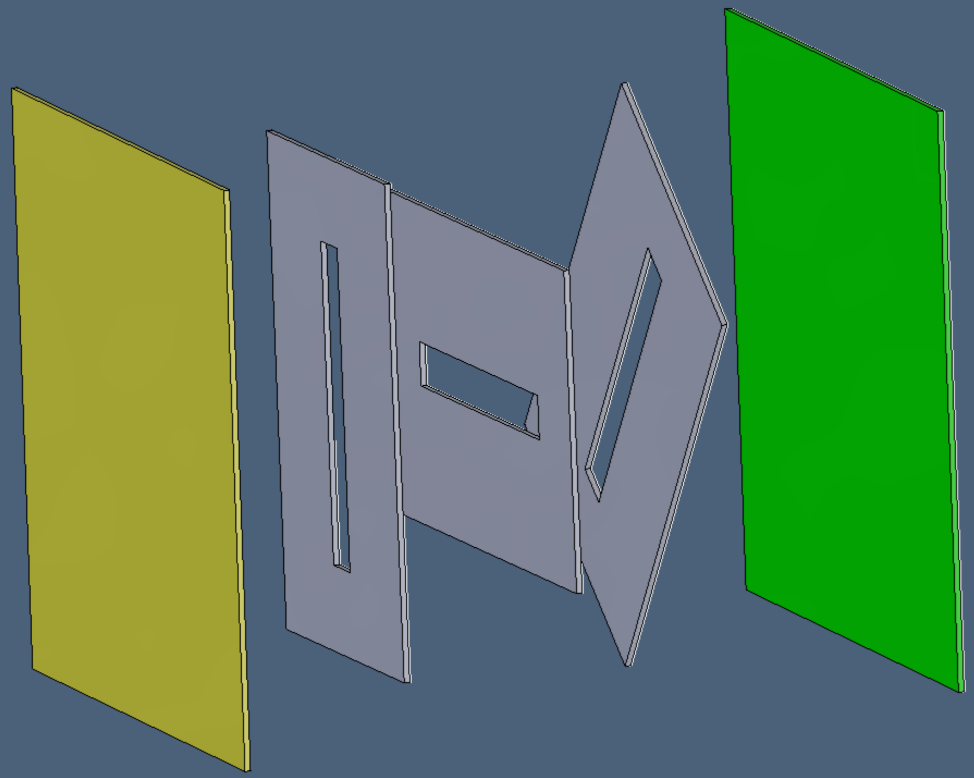
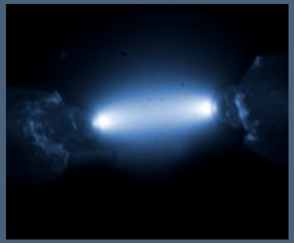
Fuzzy or strange etendue limits

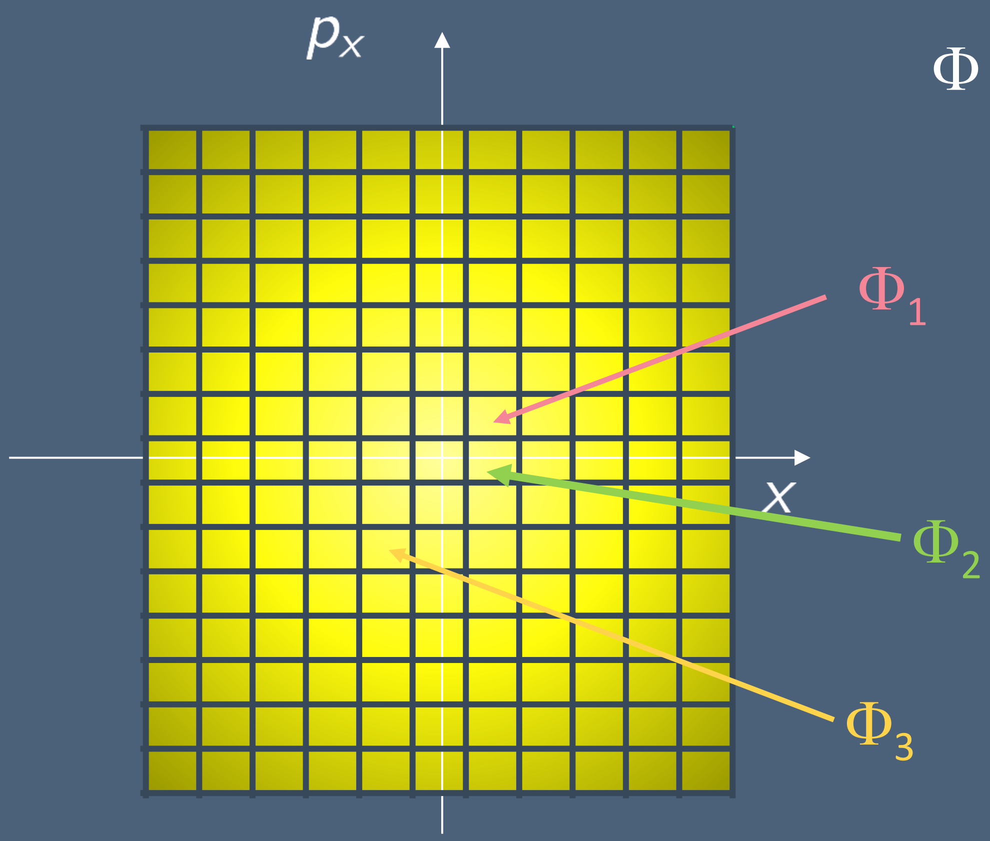




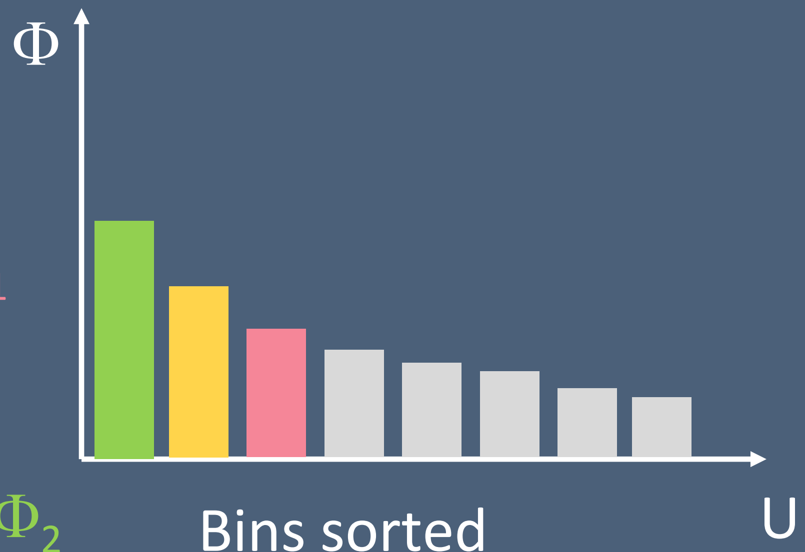


Fuzzy source handling

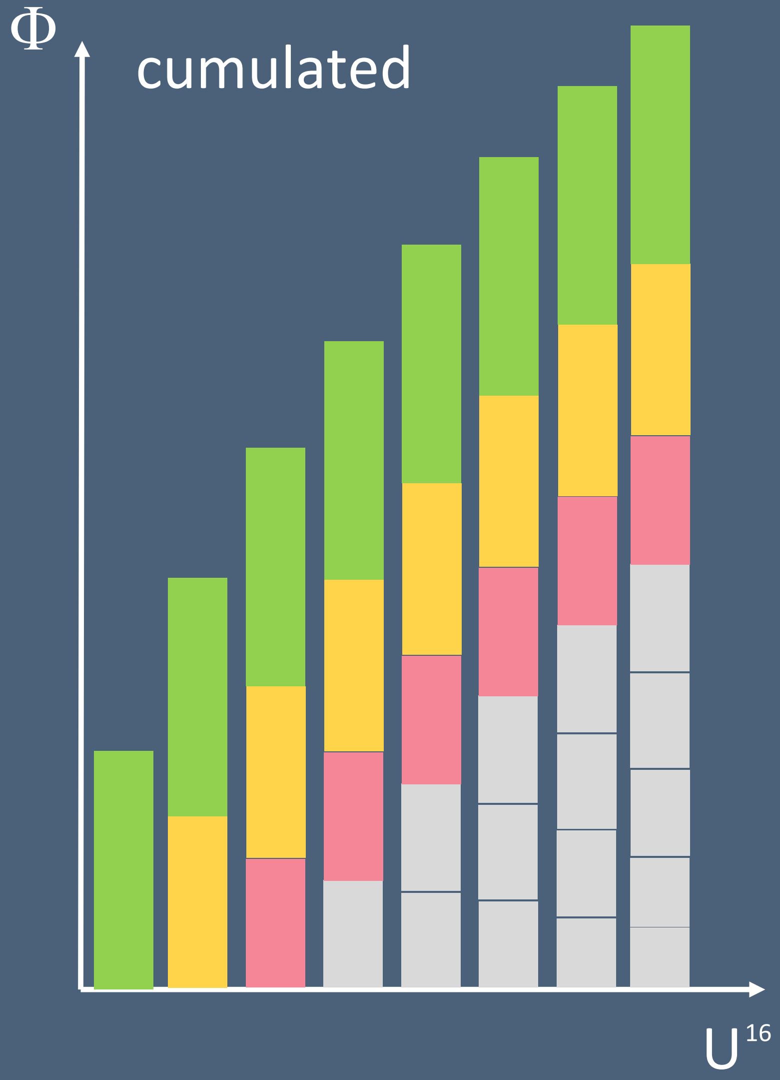
Phase space binning



Bins with flux



Bins sorted



Calculate maximum collection

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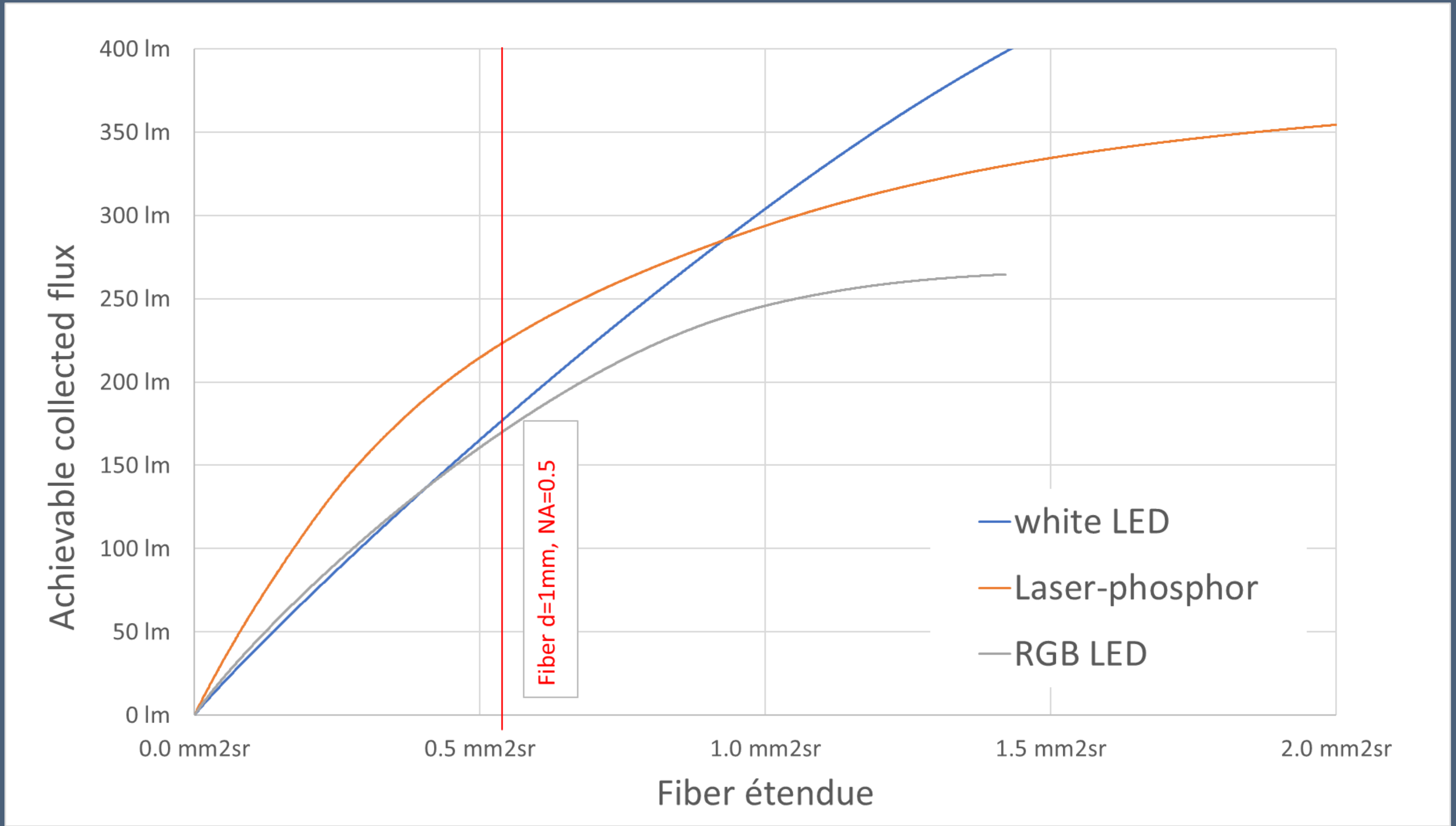
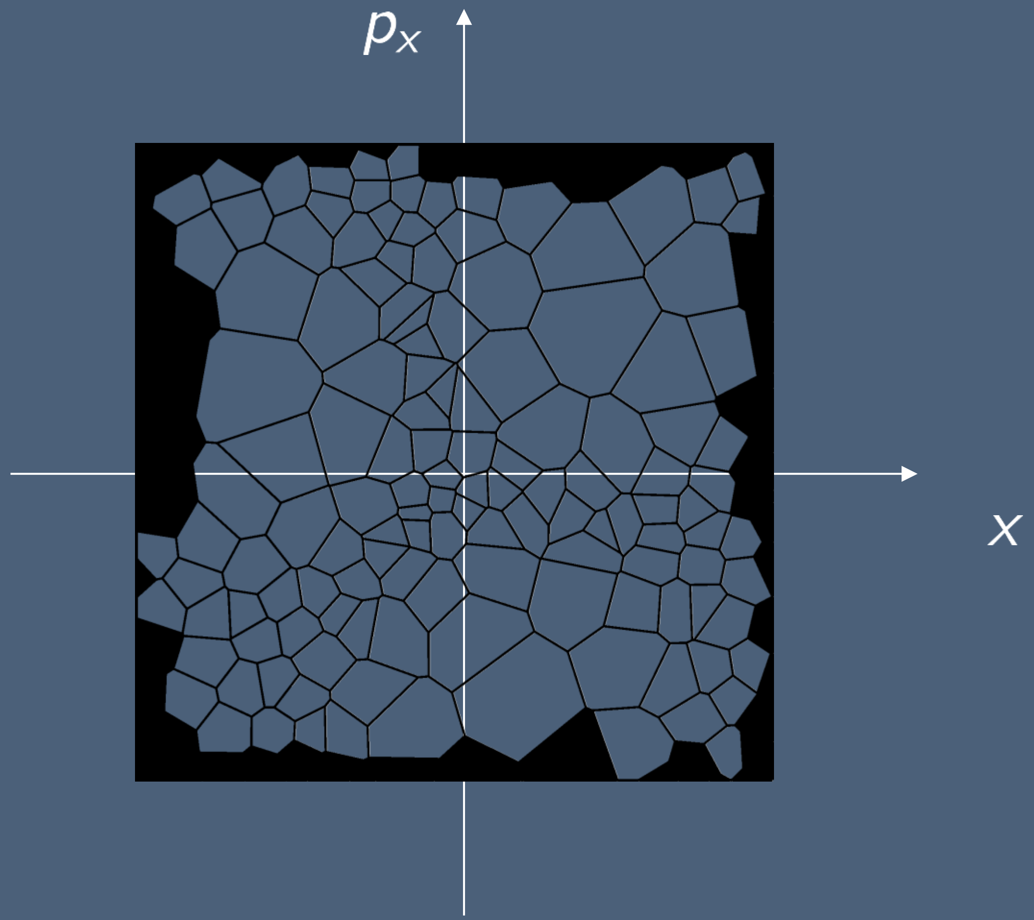
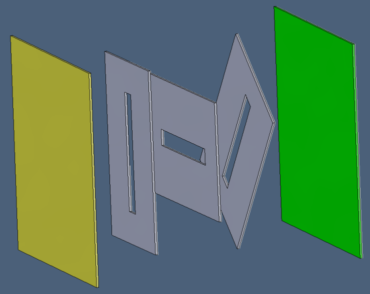


Tool to handle fuzzy distributions

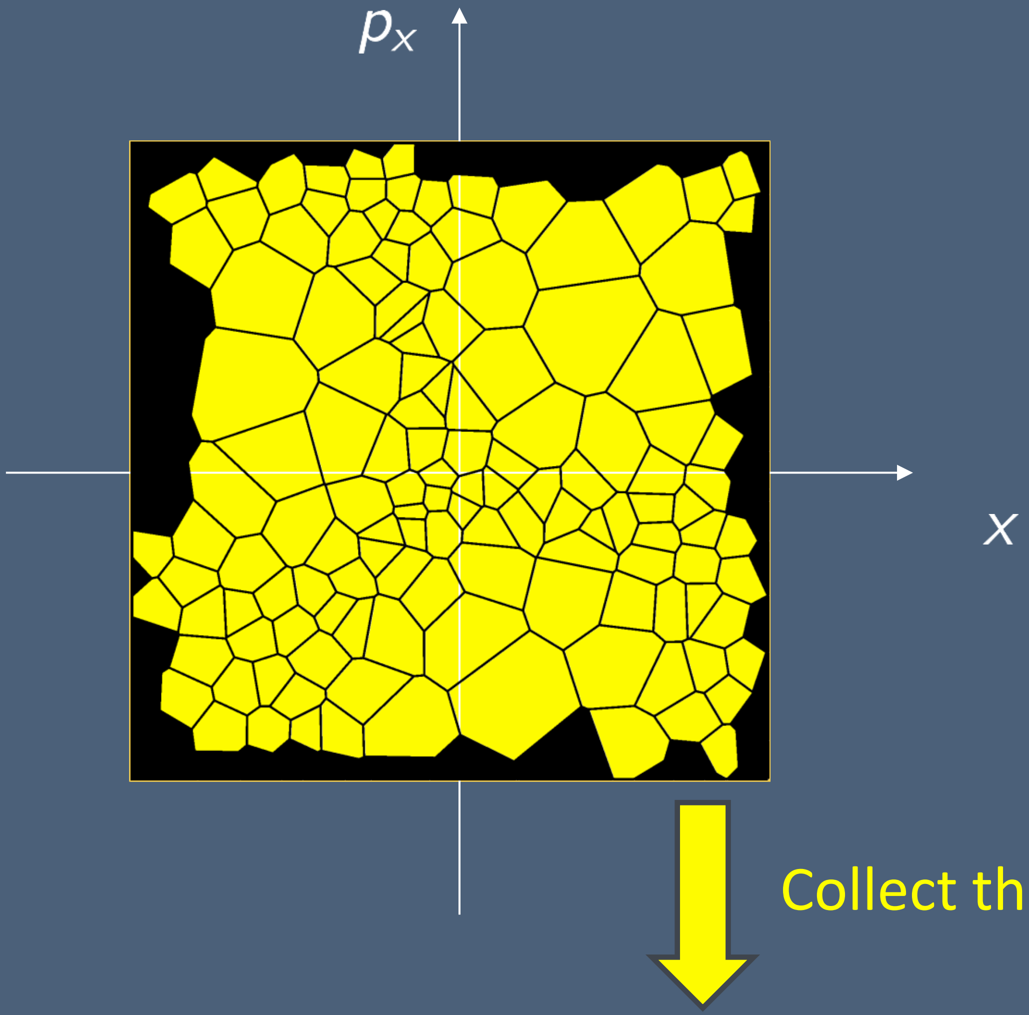
Etendue of a non-trivial system





https://www.nicepng.com/maxp/u2e6y3e6r5o0o0q8/

Etendue of non-trivial system

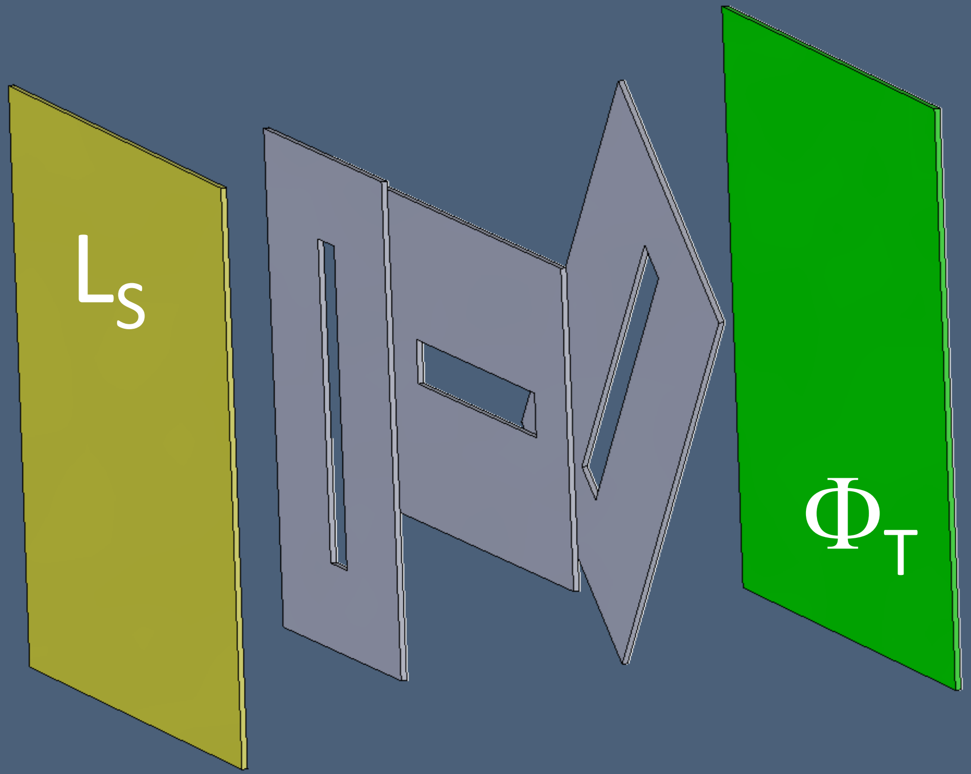


Fill the system etendue with light of constant luminance

Collect the flux

https://www.nicepng.com/maxp/u2e6y3e6r5o0o0q8/

Etendue measurement by simulation



(1) Set up
    (1) a model of the optical system
    (2) a Lambertian Emitter as source
    (3) a Receiver at target location
(2) Measure flux Φ_T
(3) Then,

$$U = \frac{\Phi_T}{L_S}$$

Etendue measurement by simulation

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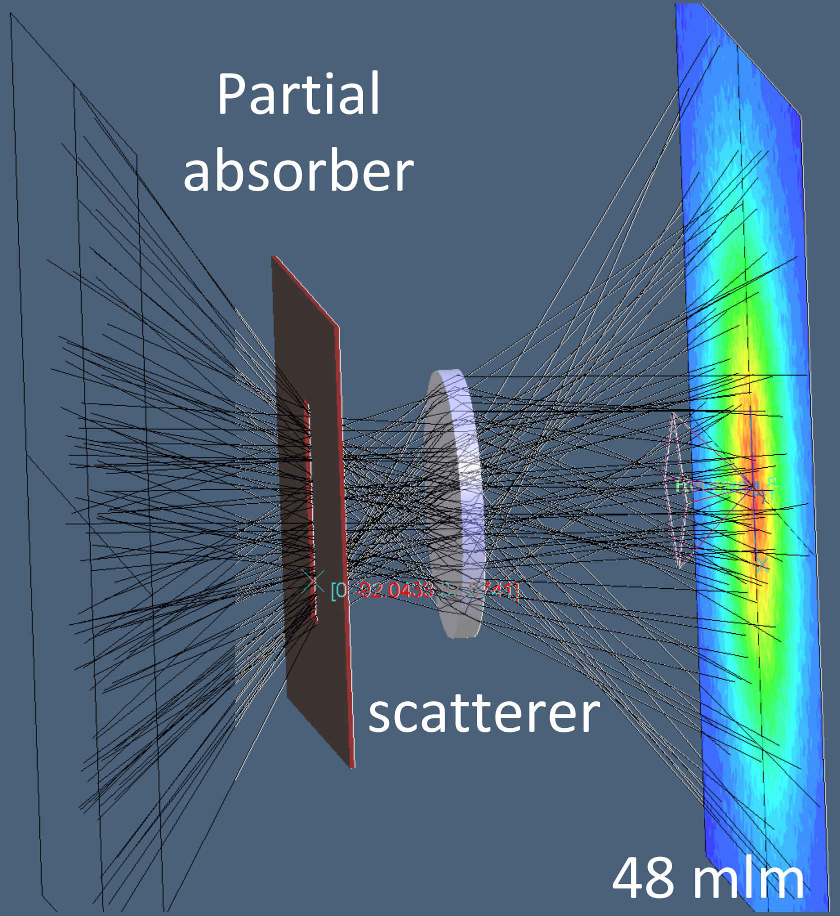
Set to 12.57 lm
(whole sphere)

→ L_S = 100 nit



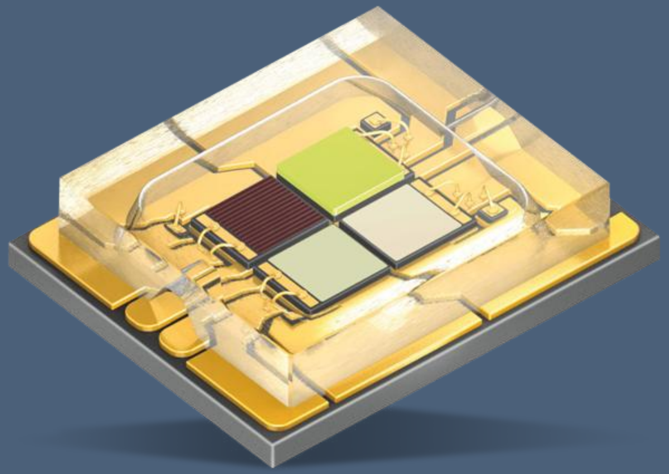
$$U = \frac{\Phi_T}{L_S}$$

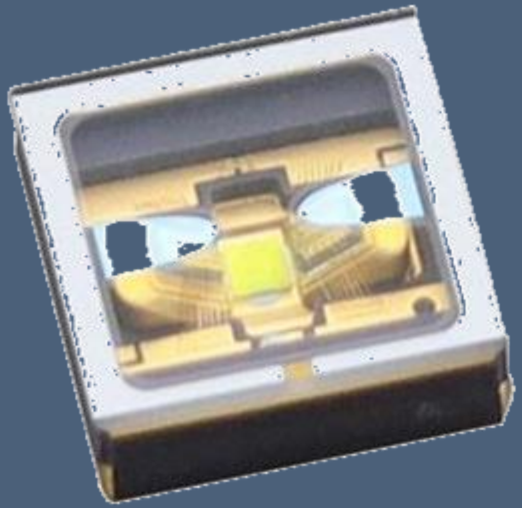
$$U = \frac{48\ mlm}{100\ nit} = 480\ \mu m^2 sr$$

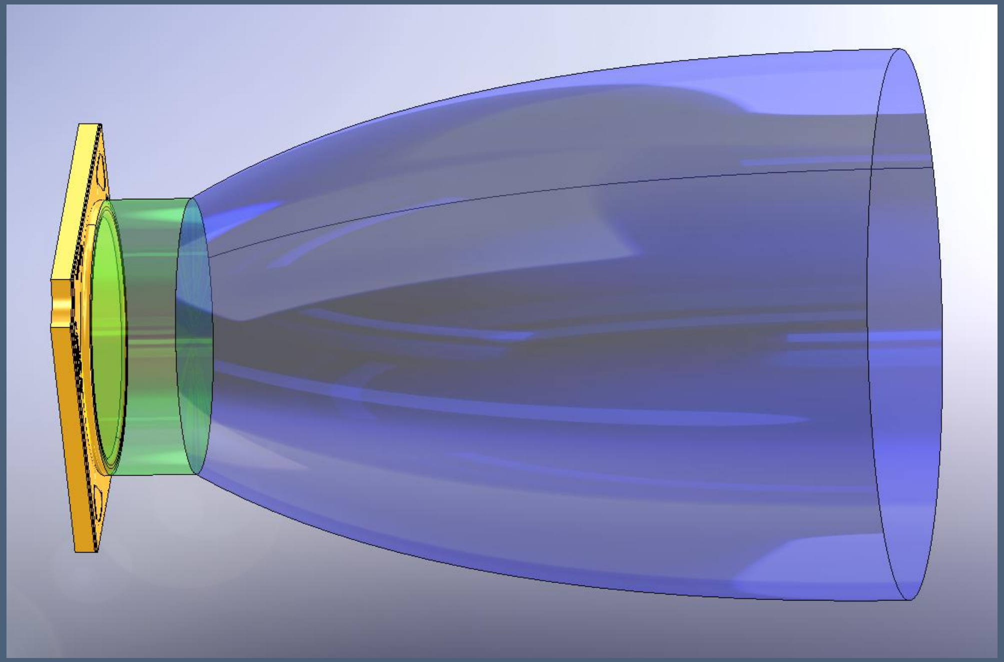
Etendue is what we measure

Application

What is the effect of a thick cover glass ?







Medium inside light tube: etendue

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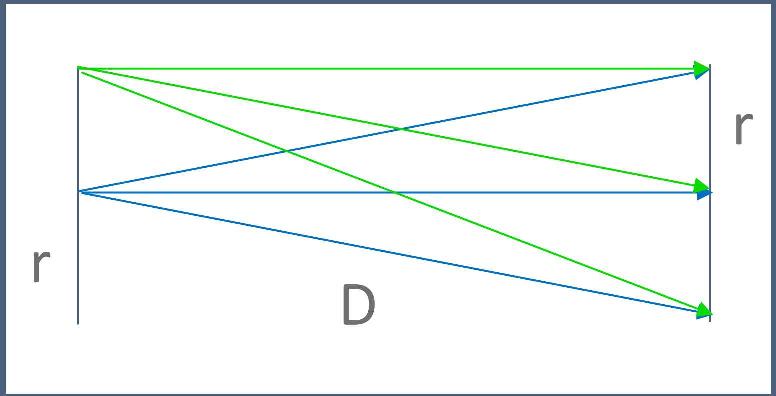
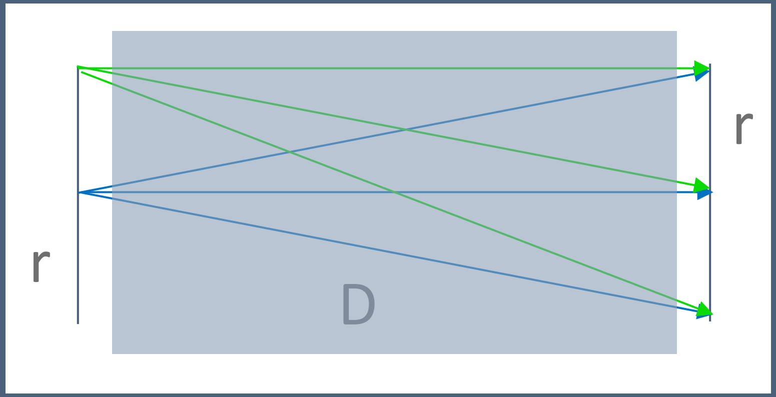
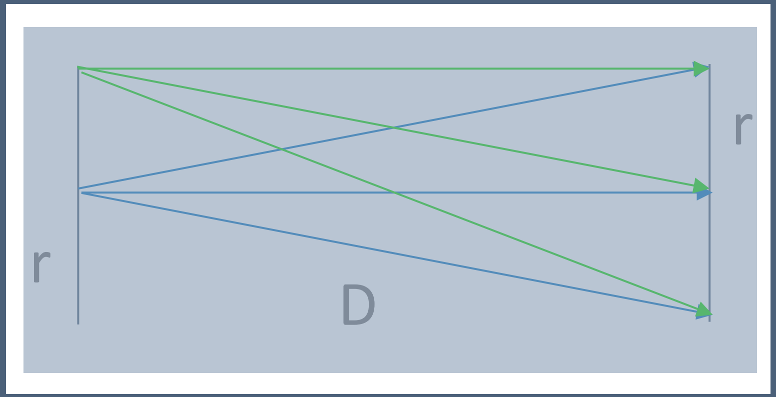
Equal
Sides
Light tube



$$U = \frac{\pi^2}{4}\left(\sqrt{(2r)^2 + D^2} - D\right)^2 sr$$

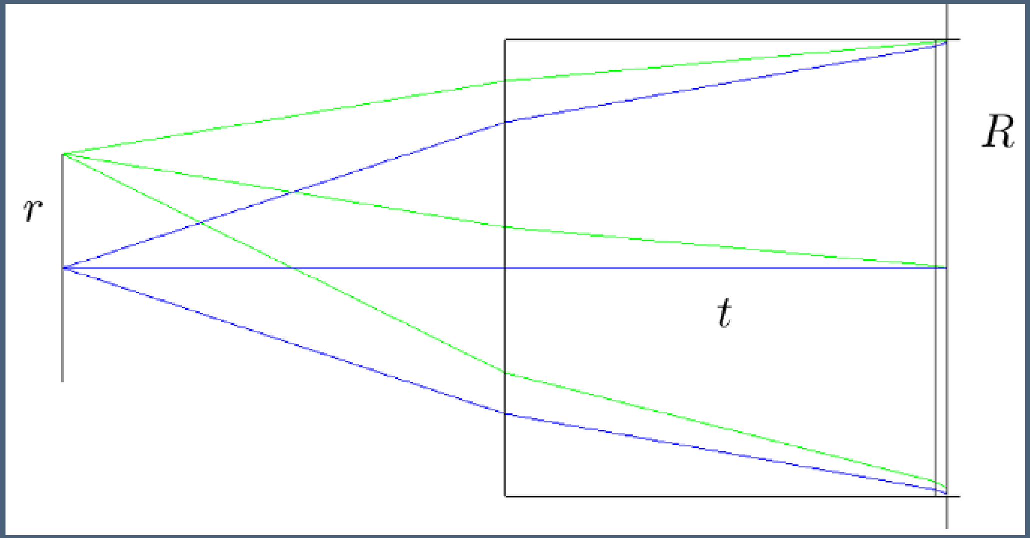




Glass (n)

Etendue of the light tube

Analytically:



t = absolute thickness

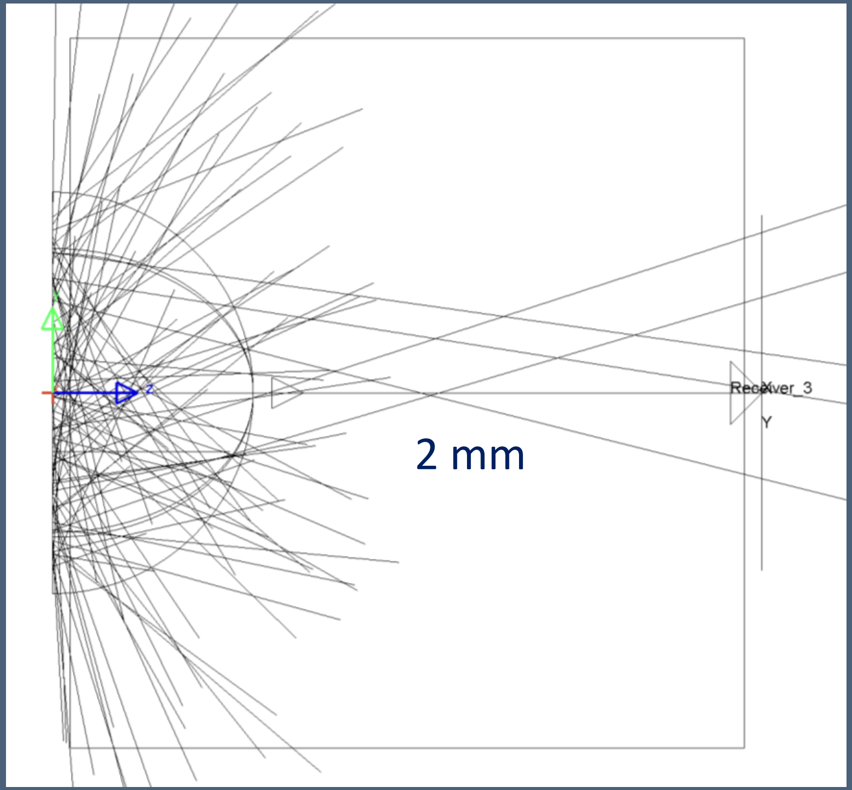
$$R = \tan\theta_1(D - t) + \tan\theta_2 t$$

$$\tan\theta_1 \approx \frac{R}{D - t + t/n}$$

$$U = (\pi r \sin\theta)^2 sr \approx \left(\frac{\pi r R}{D - t + t/n}\right)^2 sr$$

Etendue measurement

□ 1 mm², π Lm, 1 Mnit



- Source $U = \pi\, mm^2 sr$
- Light tube in air $U \approx 0.22\ mm^2 sr$ (□ by factor $(4/\pi)^2$)
- embedded in n=1.5 $U = n^2 \cdot 0.22\ mm^2 sr \approx 0.53\ mm^2 sr$
- glass inside formula t → 1 $U \approx \left(\frac{\pi r R}{D - t + t/n}\right)^2 sr = 0.53\ mm^2 sr$
- Measured:
  - Air $U \approx 0.21\ mm^2 sr$
  - Glass block $U \approx 0.48\ mm^2 sr$

Fuzzy target ?

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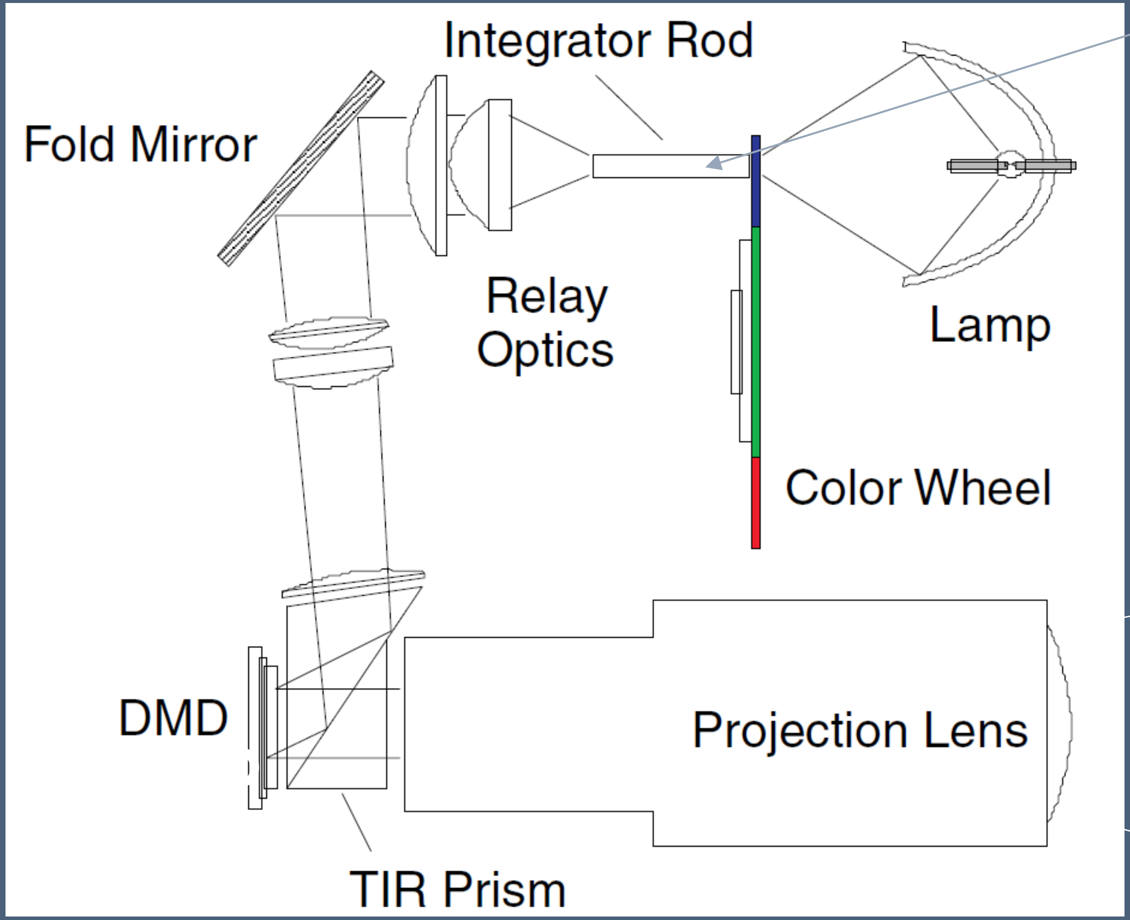
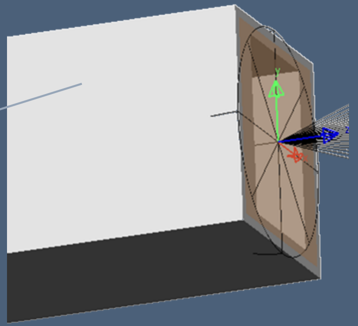
Fuzzy source, irregular system ✓

Hard edge target

Target = Receiver with system

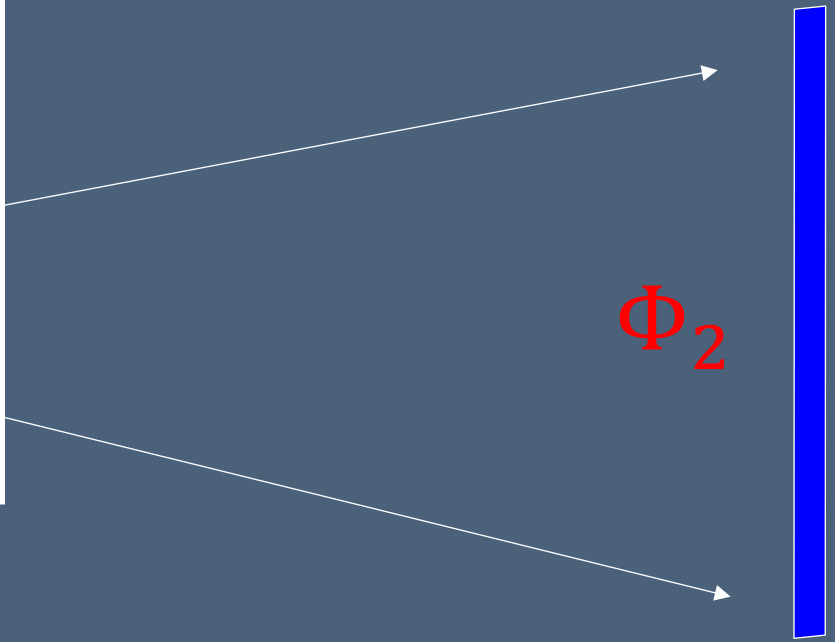
Application: Projector = Fuzzy receiver





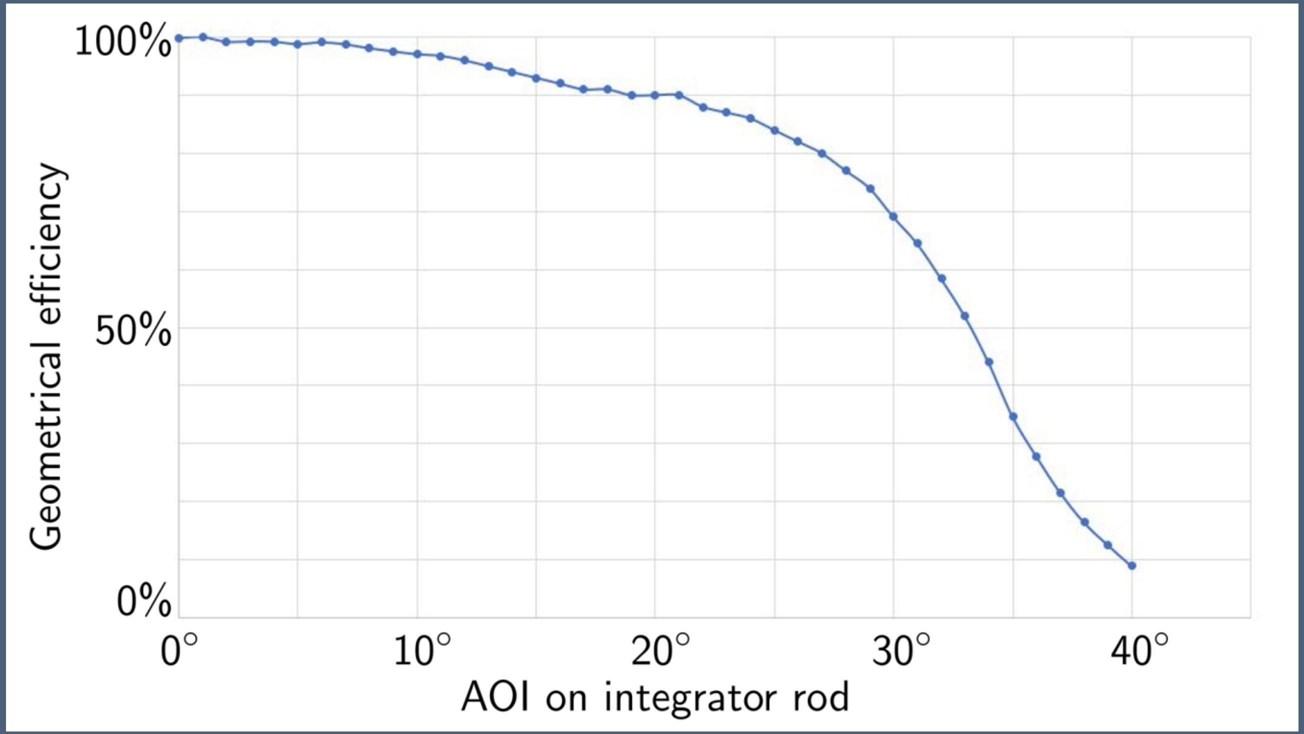
L_1

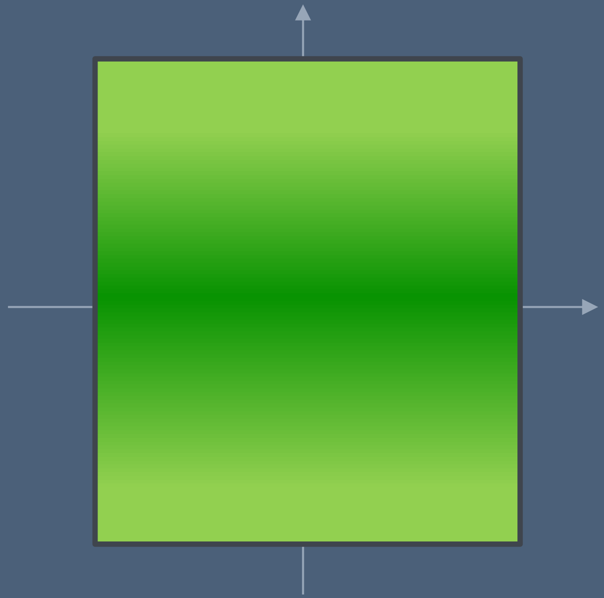
U=8mm



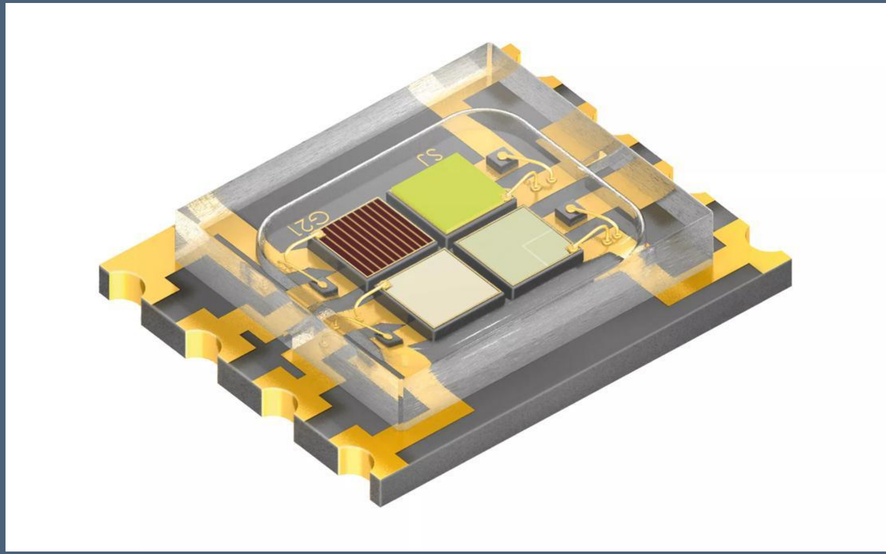
Φ_2

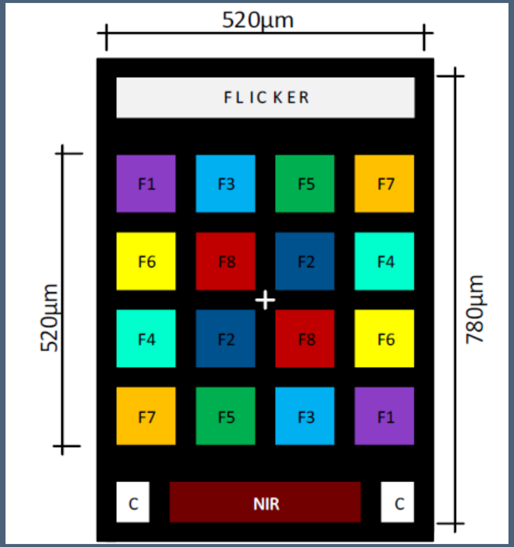
Angular receiver acceptance





Target description





Source → rayfile

Receiver → ?

Conclusion

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OPTICAL DESIGN

- We can handle objects and estimate their etendue even for fuzzy or irregular limits.
- Same etendue for
  - completely embedded and
  - almost filled light tubes.
- A cover glass is less detrimental than an air gap of the same size