The Case for Angular Extent *X*

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The large zoo of physical quantities

List of physical quantities - Wikipedia

en.wikipedia.org/wiki/List_of_physical_quantities

...tensive), their transformation properties (i.e. whether the quantity is a scalar, vector, matrix or tensor), and whether the quantity is conserved.

Scalar [edit]

Main category: Scalar physical quantities

Scalar quantities

Name	Symbol	Description	SI unit	Quantity dimension	Comments
absorbed dose rate	*D*	Absorbed dose received per unit of time	Gy/s	$L^2 T^{-3}$	
action	*S*	Momentum of particle multiplied by distance travelled	J/Hz	$M L^2 T^{-1}$	
amount of substance	*n*	The quantity proportional to the number of particles in a sample, with the Avogadro constant as the proportionality constant	mole (mol)	N	ISQ base quantity, extensive
angular acceleration	ω_a	Change in angular velocity per unit time	rad/s^2	T^{-2}	

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What makes a good quantity?

$$E_{\text{kin}} = \frac{m}{2} v^2$$

$$E_{\text{kin}} + E_{\text{pot}} + E_{\text{thermal}} + \ldots = \text{const}$$

$$\cancel{G_{\text{special}} = \frac{m^2}{3} v^3}$$

Robin Higgins, pixabay

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What makes a good quantity?

$$E_{\text{kin}} = \frac{m}{2} v^2$$

$$E_{\text{kin}} + E_{\text{pot}} + E_{\text{thermal}} + \ldots = \text{const}$$

- Shortens lengthy expressions
- Features prominently in laws of Nature
- Has an intuitive name
- Unique, descriptive symbol

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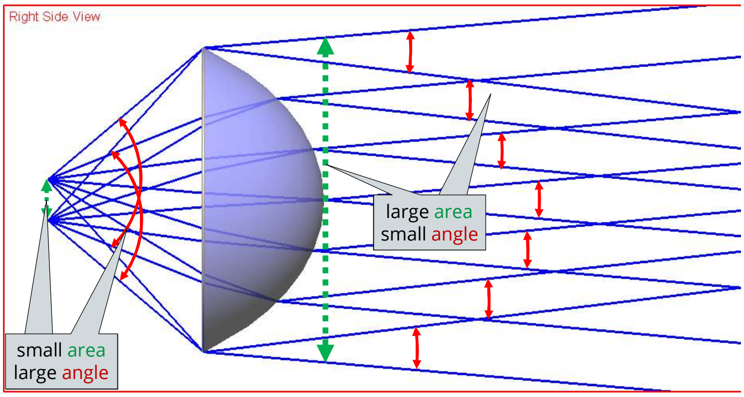
Etendue *U*



„Etendue" = area · solid angle = const ?

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Infinitesimal etendue and the cos(θ) factor

Projected small area element to Small solid angle element

=

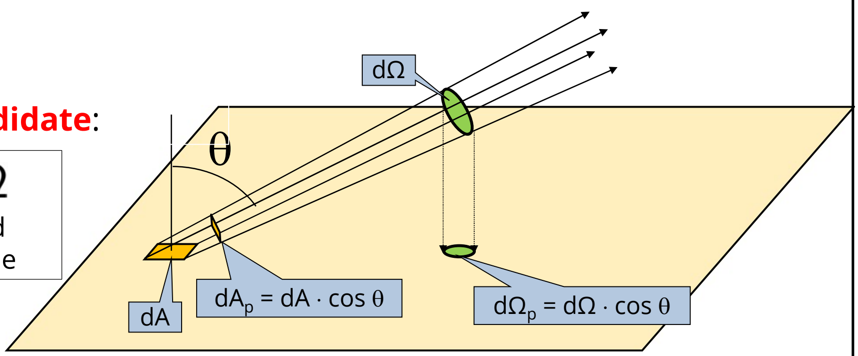
Small area element to **Projected** small solid angle element

Etendue building block **candidate**:

$$dU = dA \cdot \cos\theta \cdot d\Omega$$

Projected area

Projected solid angle



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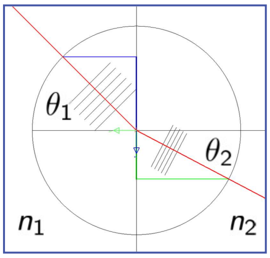
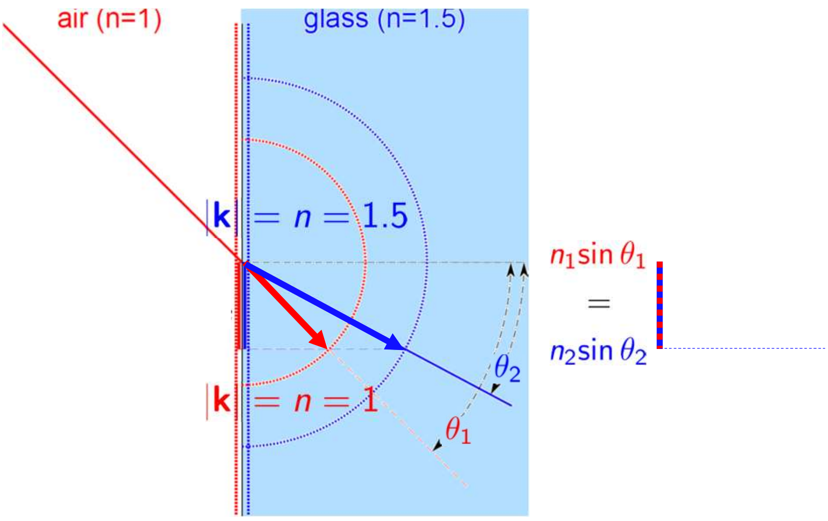
What about Snell's law?

JMO Illumination Optics — ILLUMINATIO SOLUTIONS OPTICAL DESIGN



$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



Infinitesimal etendue

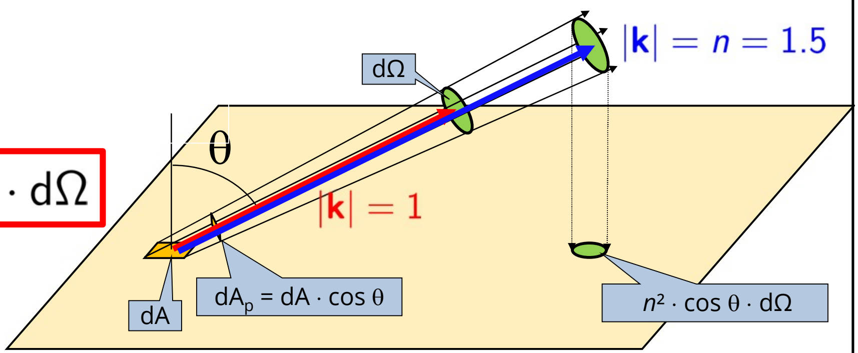
JMO Illumination Optics — ILLUMINATIO SOLUTIONS OPTICAL DESIGN

$$dX = n^2 \cdot \cos\theta \cdot d\Omega \qquad dU = dA\, dX$$

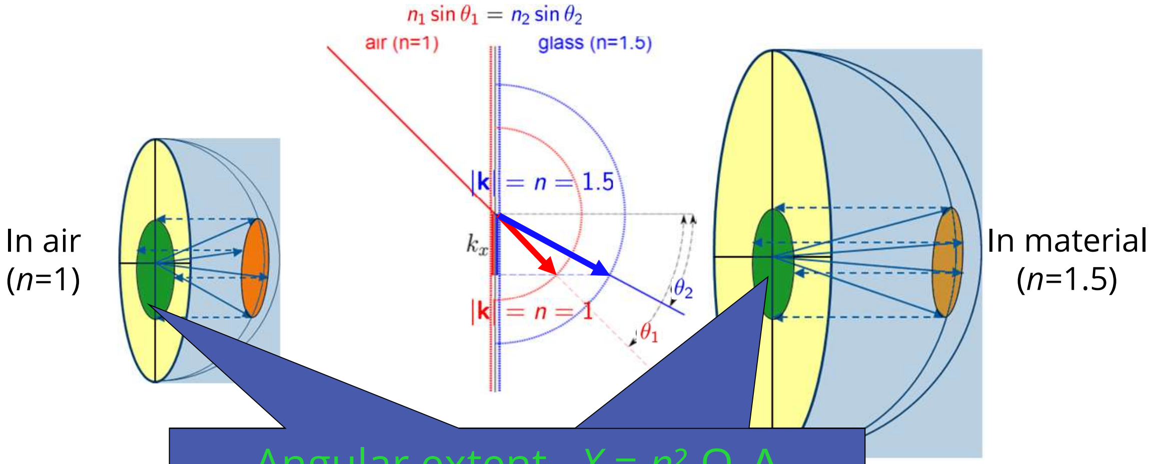
Etendue building block:

$$dU = dA \cdot \boxed{n^2 \cdot \cos\theta \cdot d\Omega}$$

Infinitesimal angular extent dX

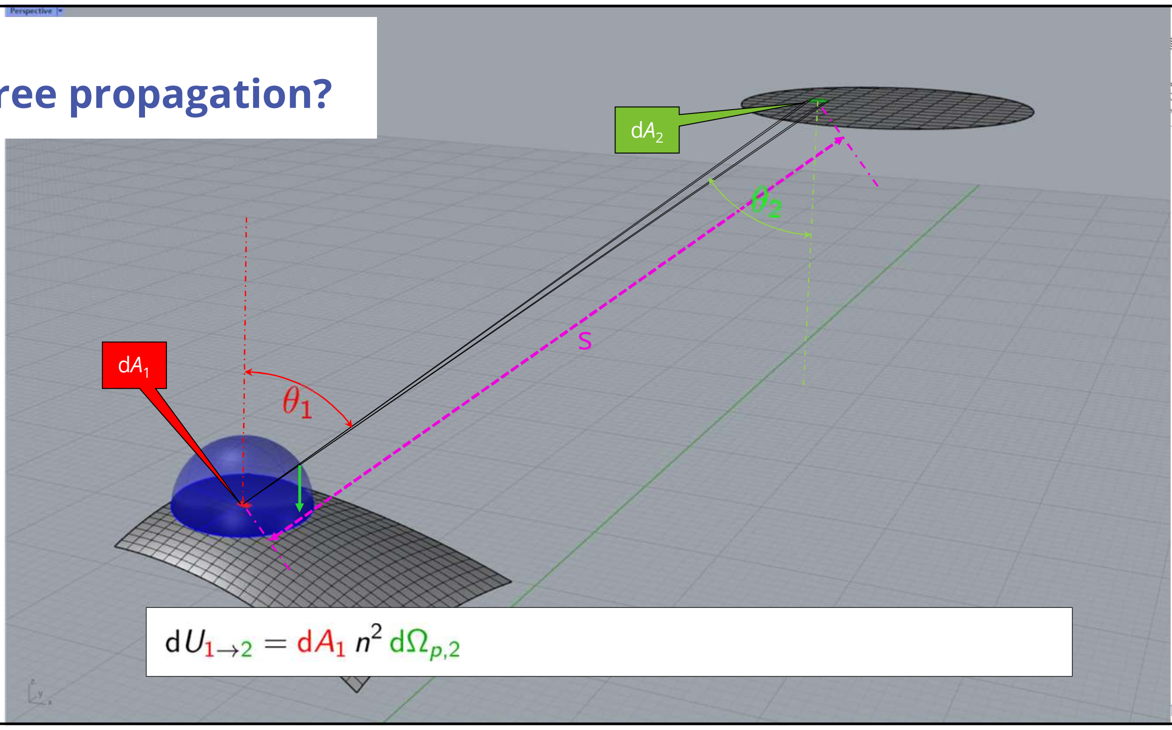


Refraction conserves angular extent, thus étendue

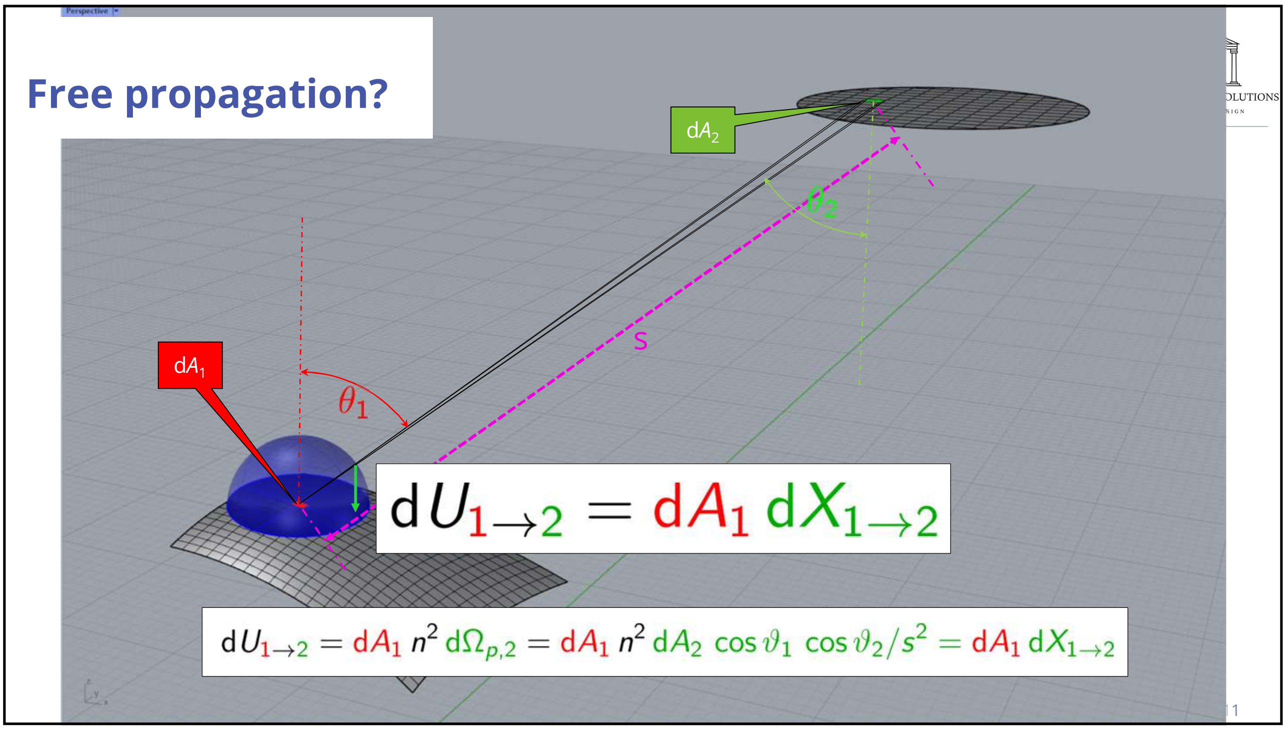


Angular extent $X = n^2\,\Omega_p A$
same before and after refraction

Free propagation?



$$\mathrm{d}U_{1\to 2} = \mathrm{d}A_1\, n^2\, \mathrm{d}\Omega_{p,2}$$


Free propagation?
dA_2
ϑ_2
s
dA_1
θ_1
$\mathrm{d}U_{1\to2} = \mathrm{d}A_1\, \mathrm{d}X_{1\to2}$
$\mathrm{d}U_{1\to2} = \mathrm{d}A_1\, n^2\, \mathrm{d}\Omega_{p,2} = \mathrm{d}A_1\, n^2\, \mathrm{d}A_2\, \cos\vartheta_1\, \cos\vartheta_2/s^2 = \mathrm{d}A_1\, \mathrm{d}X_{1\to2}$

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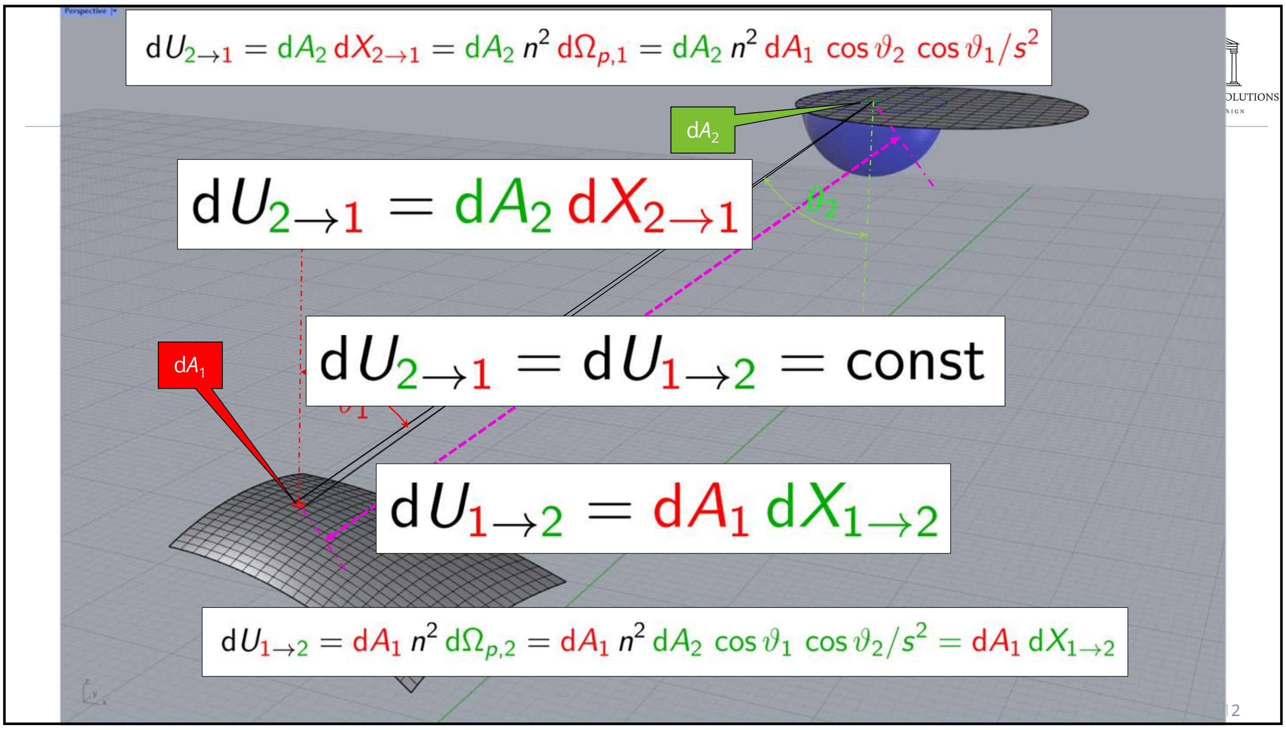
$\mathrm{d}U_{2\to1} = \mathrm{d}A_2\, \mathrm{d}X_{2\to1} = \mathrm{d}A_2\, n^2\, \mathrm{d}\Omega_{p,1} = \mathrm{d}A_2\, n^2\, \mathrm{d}A_1\, \cos\vartheta_2\, \cos\vartheta_1/s^2$
dA_2
$\mathrm{d}U_{2\to1} = \mathrm{d}A_2\, \mathrm{d}X_{2\to1}$
ϑ_2
dA_1
$\mathrm{d}U_{2\to1} = \mathrm{d}U_{1\to2} = \mathrm{const}$
$\mathrm{d}U_{1\to2} = \mathrm{d}A_1\, \mathrm{d}X_{1\to2}$
$\mathrm{d}U_{1\to2} = \mathrm{d}A_1\, n^2\, \mathrm{d}\Omega_{p,2} = \mathrm{d}A_1\, n^2\, \mathrm{d}A_2\, \cos\vartheta_1\, \cos\vartheta_2/s^2 = \mathrm{d}A_1\, \mathrm{d}X_{1\to2}$

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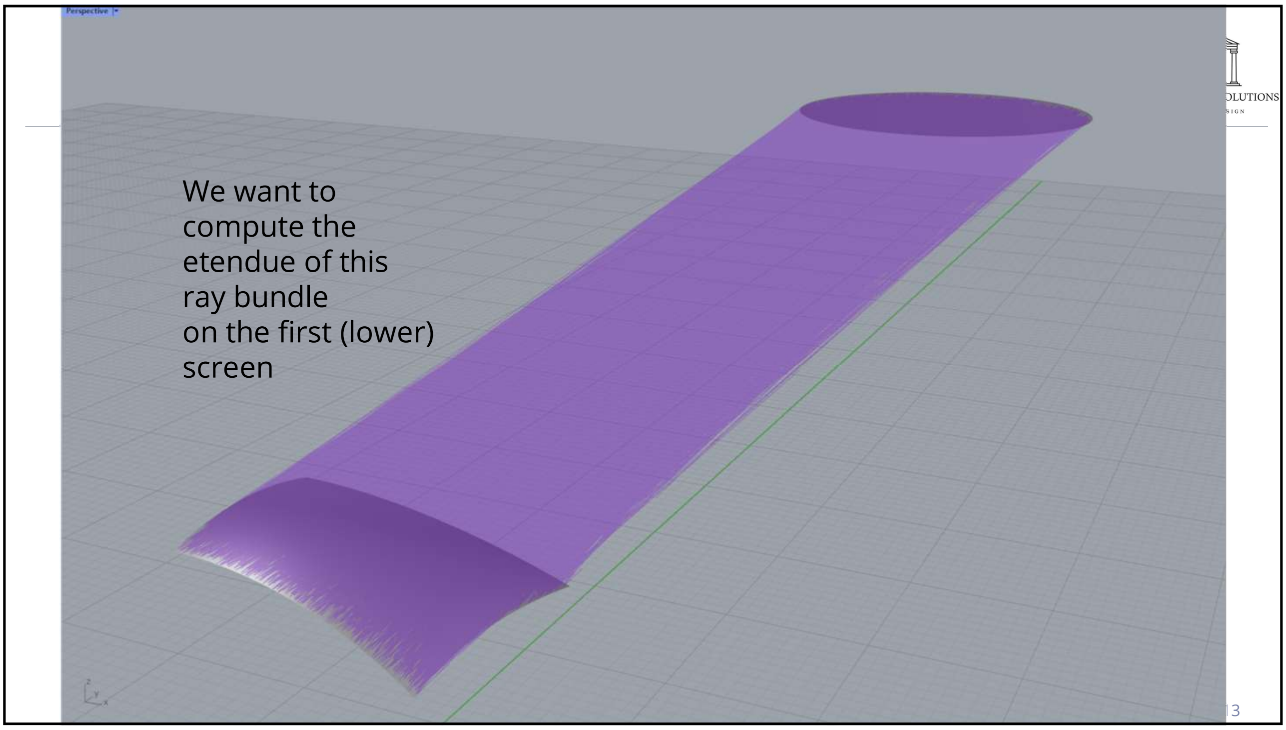
We want to
compute the
etendue of this
ray bundle
on the first (lower)
screen

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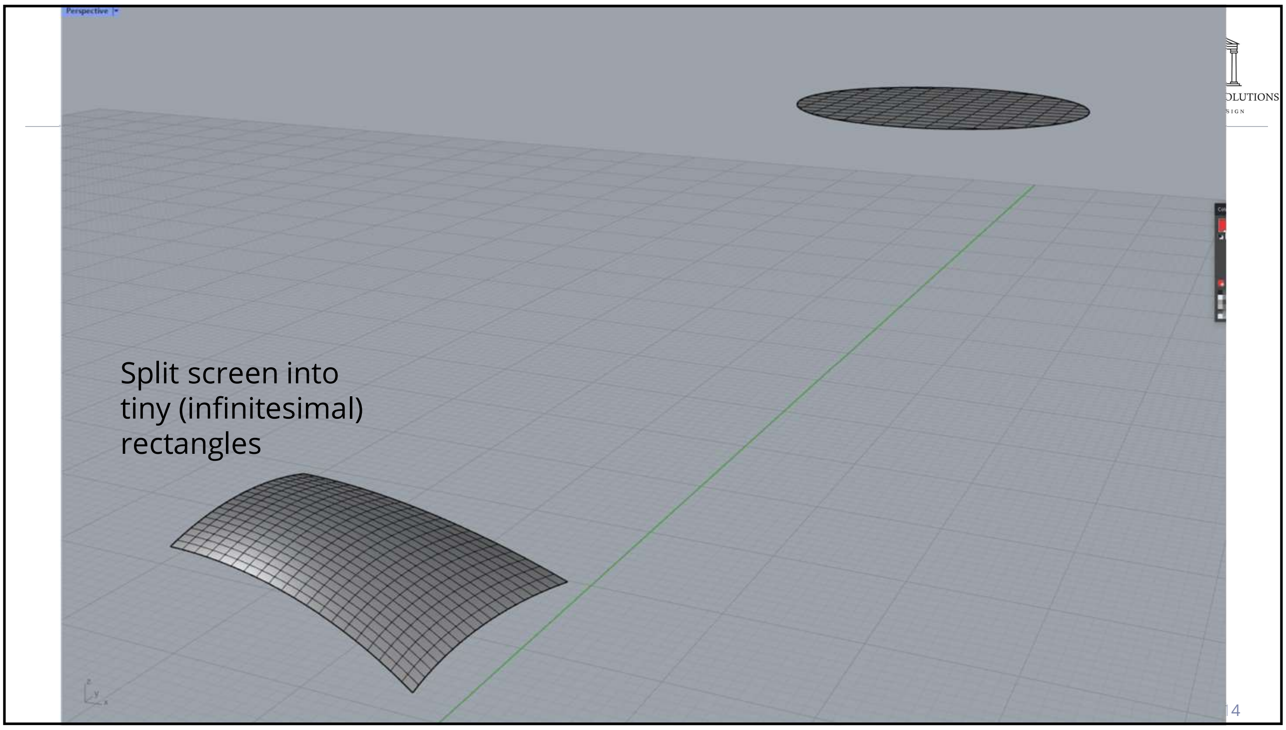
Split screen into
tiny (infinitesimal)
rectangles

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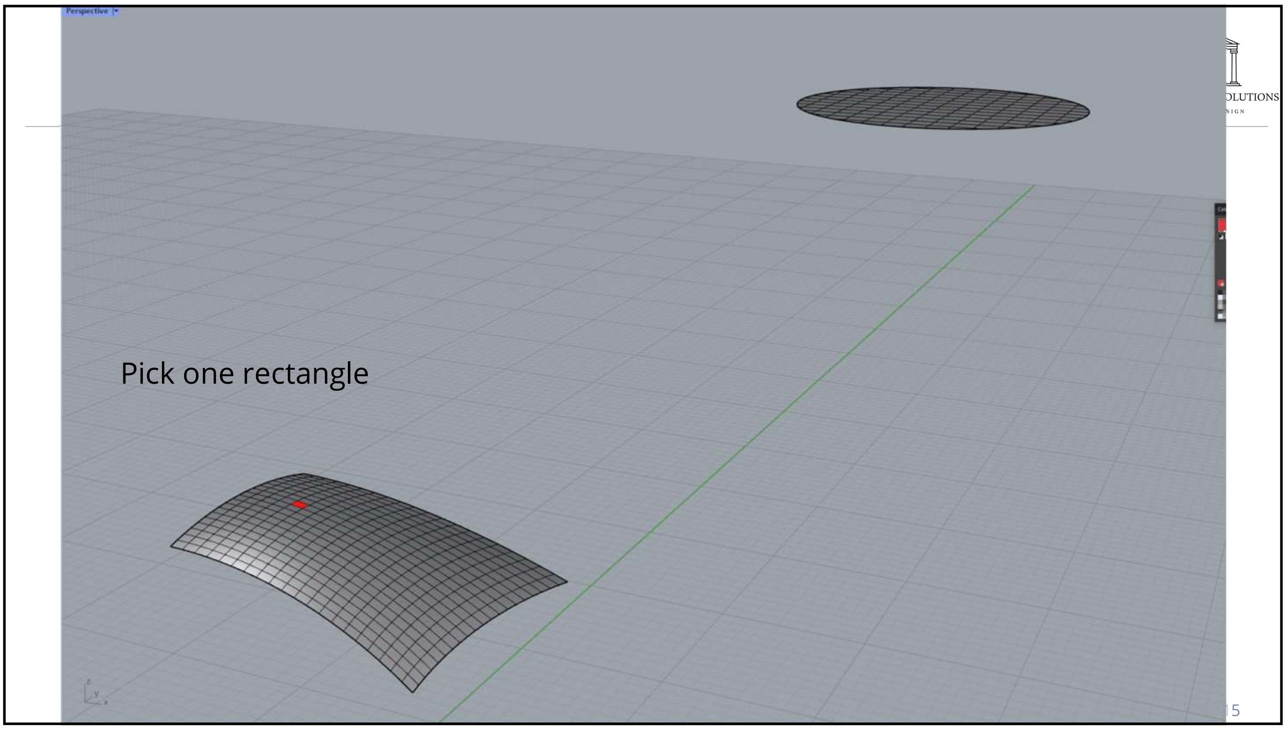

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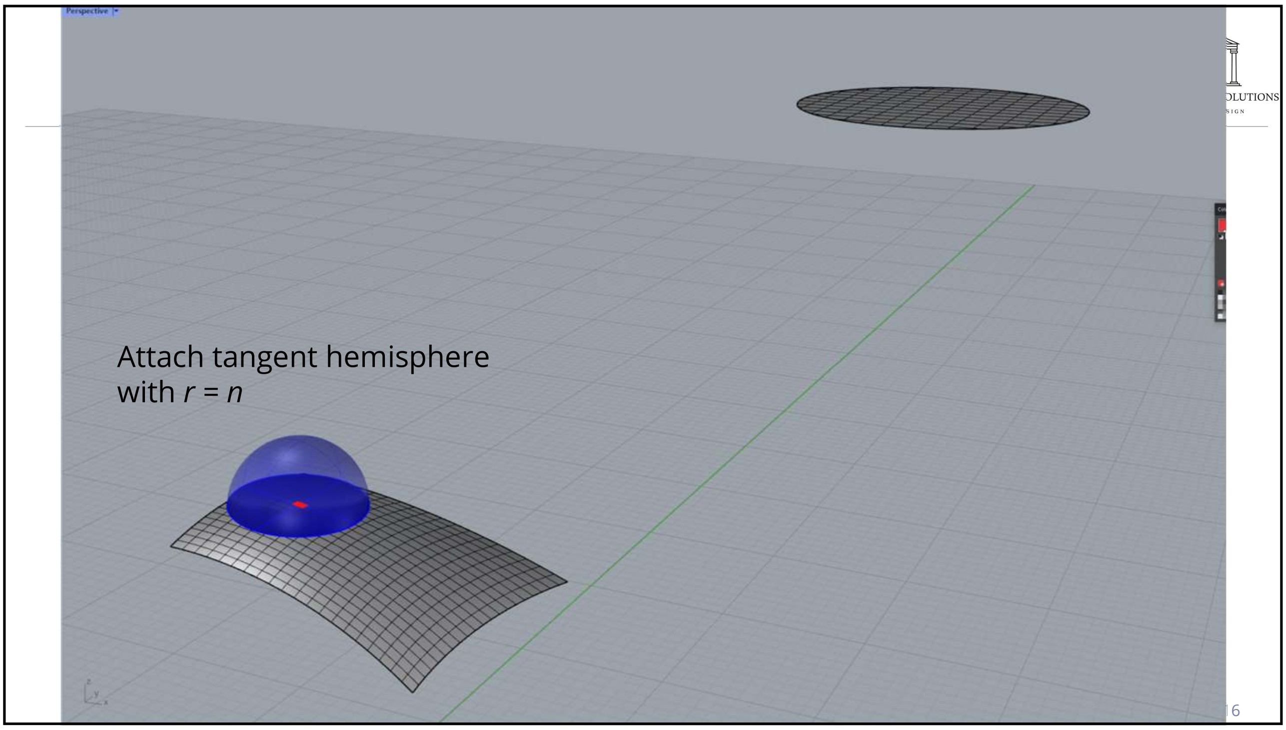

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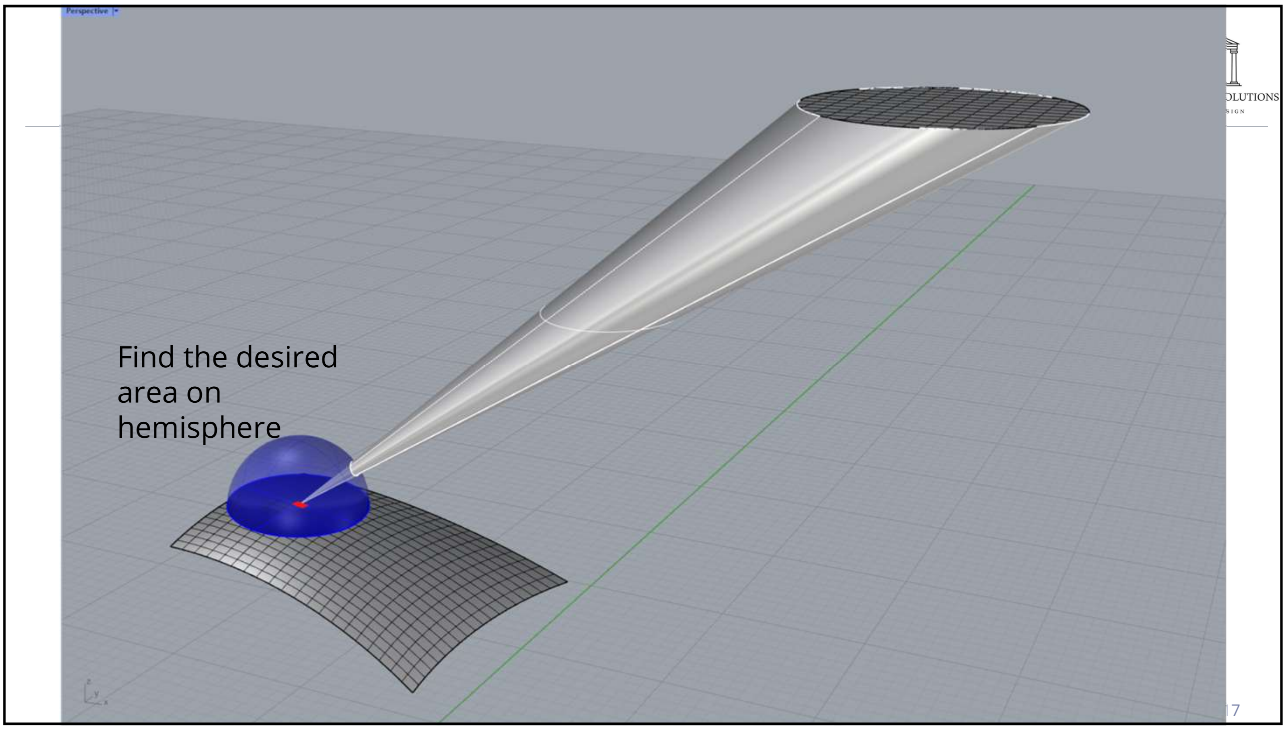

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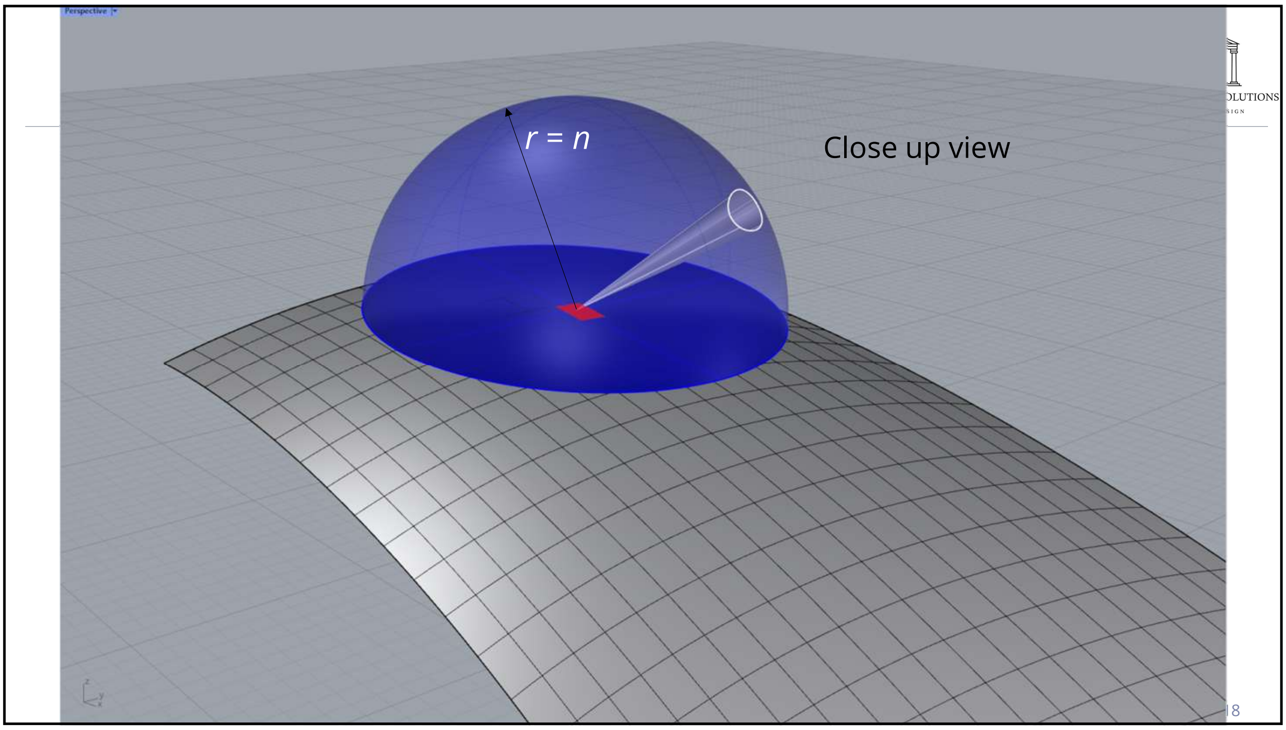

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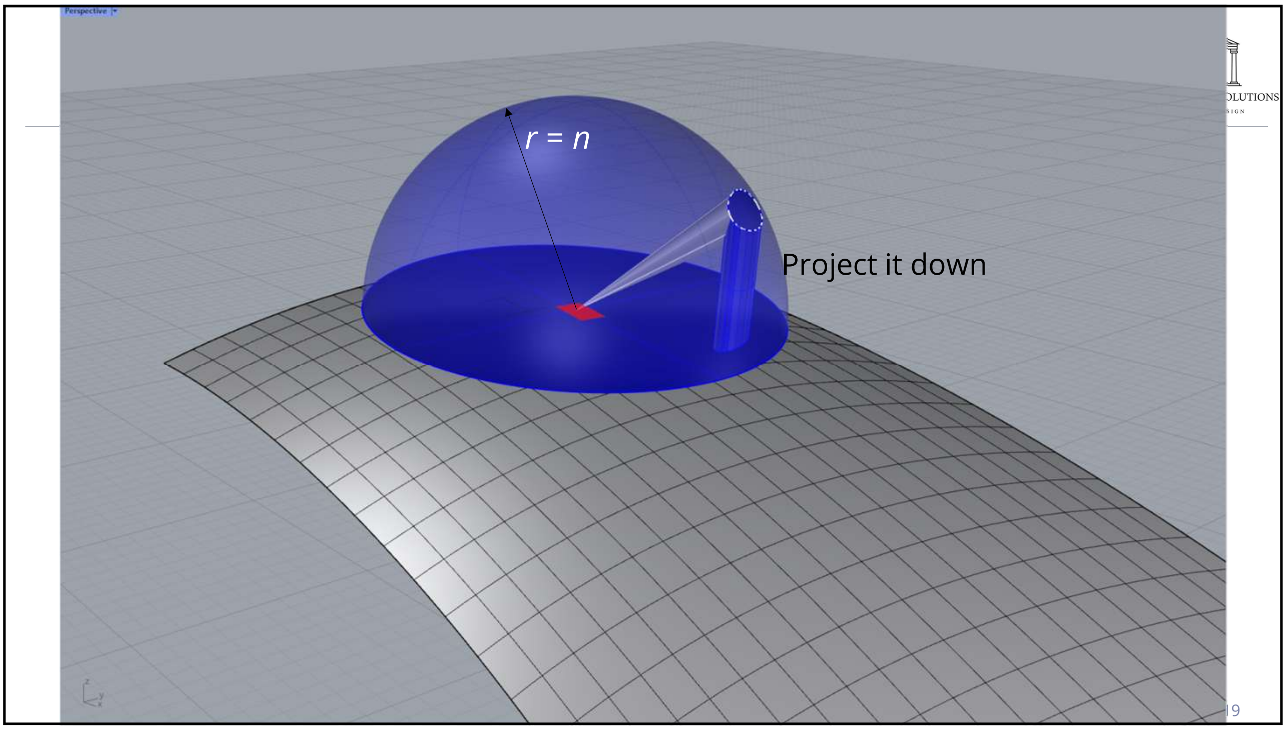
$r = n$
Project it down
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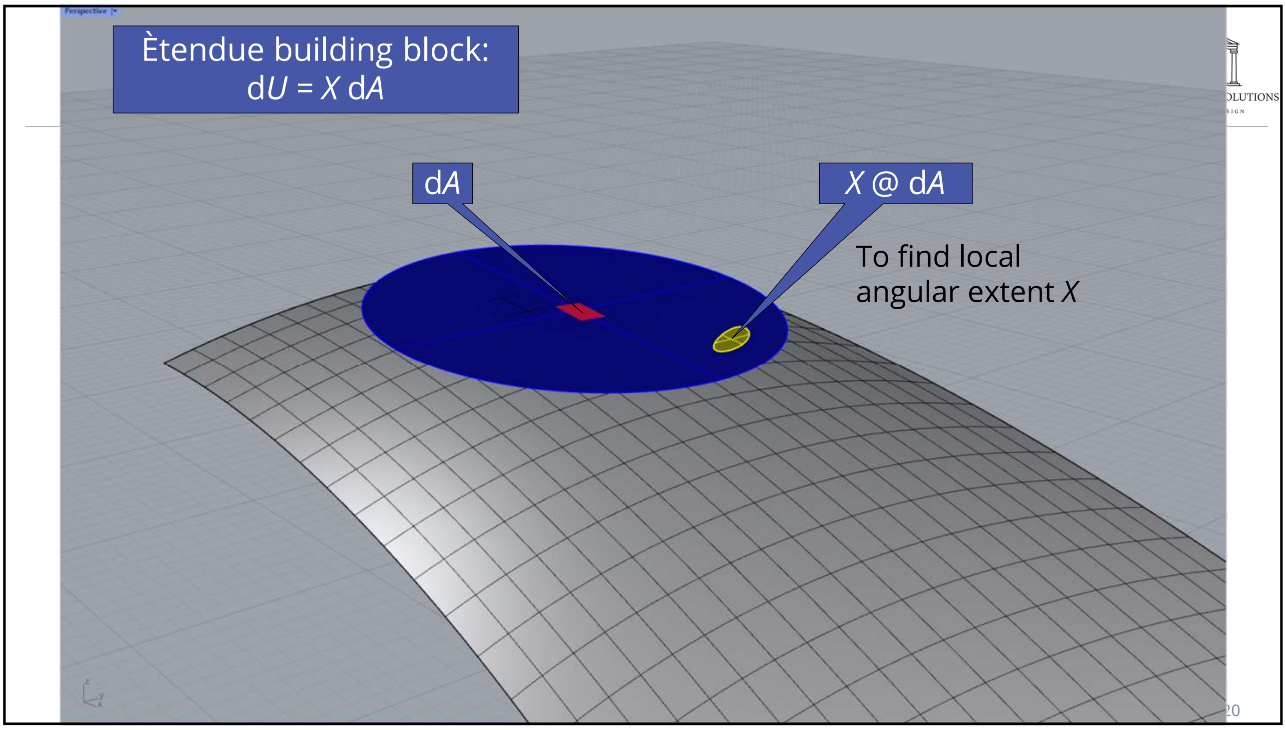
Ètendue building block:
dU = X dA
dA
X @ dA
To find local
angular extent X
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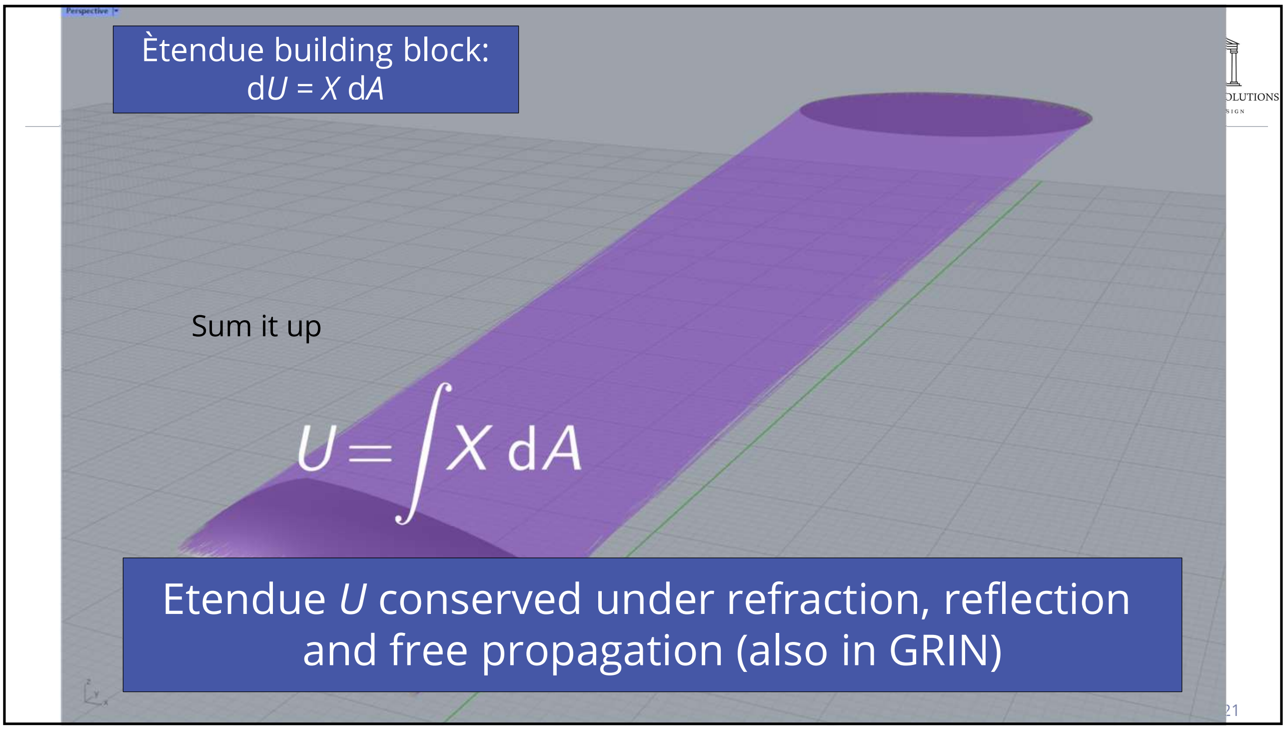
Ètendue building block:
$dU = X\,dA$
Sum it up
$U = \int X\,dA$
Etendue U conserved under refraction, reflection and free propagation (also in GRIN)

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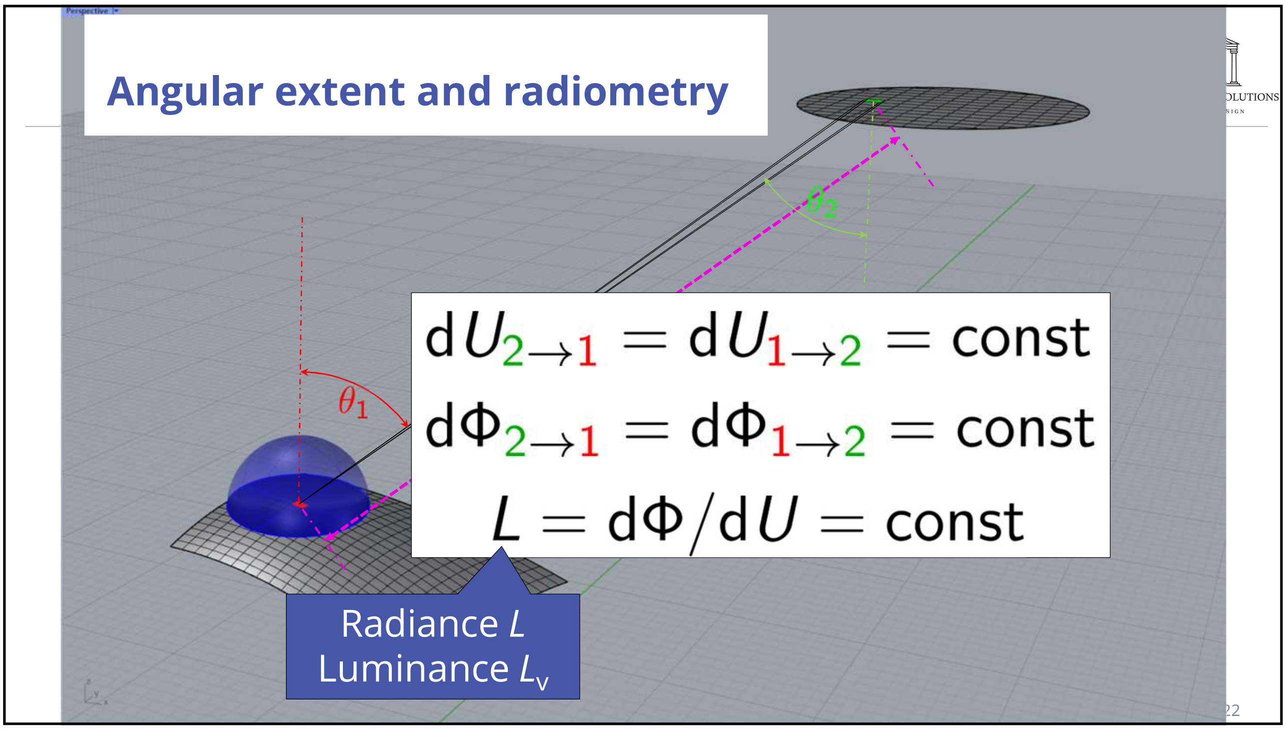
Angular extent and radiometry
θ_2
θ_1
$dU_{2\to1} = dU_{1\to2} = \text{const}$
$d\Phi_{2\to1} = d\Phi_{1\to2} = \text{const}$
$L = d\Phi / dU = \text{const}$
Radiance L
Luminance L_v

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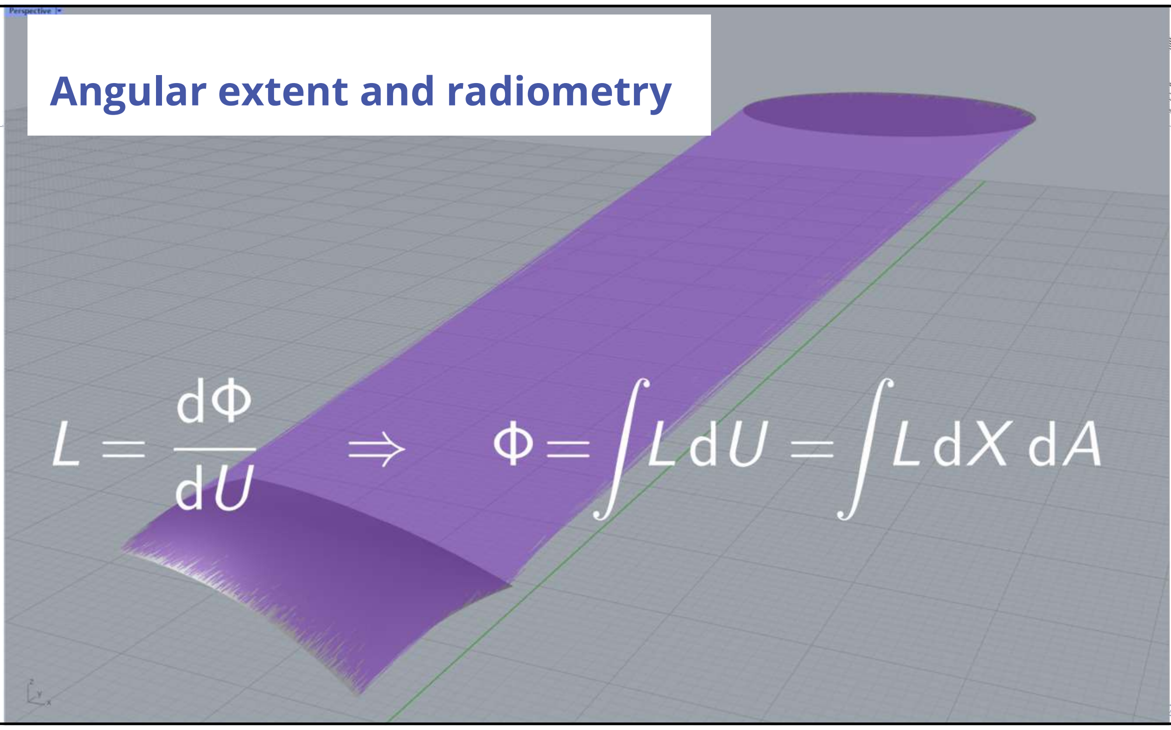
Radiance constant along rays

Angular extent and radiometry



$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}U} \quad \Rightarrow \quad \Phi = \int L\,\mathrm{d}U = \int L\,\mathrm{d}X\,\mathrm{d}A$$

Illuminance

Flux per area, no matter from where:
a function of location (*x, y*)

$$E = \frac{\mathrm{d}\Phi}{\mathrm{d}A} \quad \Leftrightarrow \quad \Phi = \int E\,\mathrm{d}A$$

Radiometric:	
Name	Irradiance
Symbol	E
Physical unit	W/m²
Photometric:	
Name	Illuminance
Symbol	E_v
Physical unit	Lux (lx = lm/m²)

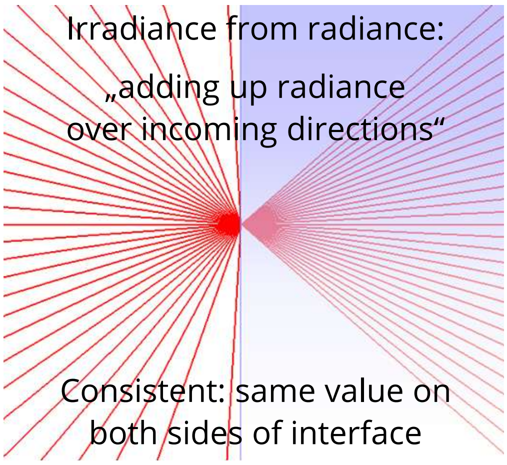
Irradiance from angular extent

$$\Phi = \int E\,\mathrm{d}A$$ from irradiance definition

$$\Phi = \int \int L\,\mathrm{d}X\,\mathrm{d}A$$ from radiance definition

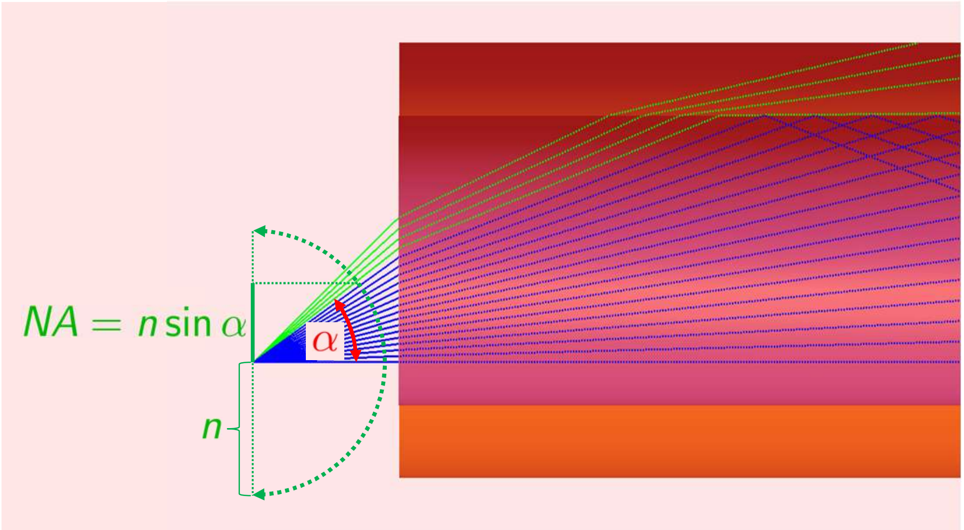
$$\Phi = \int \int L\,n^2 \cos\theta\,\mathrm{d}\Omega\,\mathrm{d}A$$

$$\Phi = \int \int L\,n^2 \cos\theta \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi\,\mathrm{d}A$$



$$E = \int L\,\mathrm{d}X = \int L\,n^2 \cos\theta\,\mathrm{d}\Omega = \int L\,n^2 \cos\theta \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi$$

From angle to numerical aperture…



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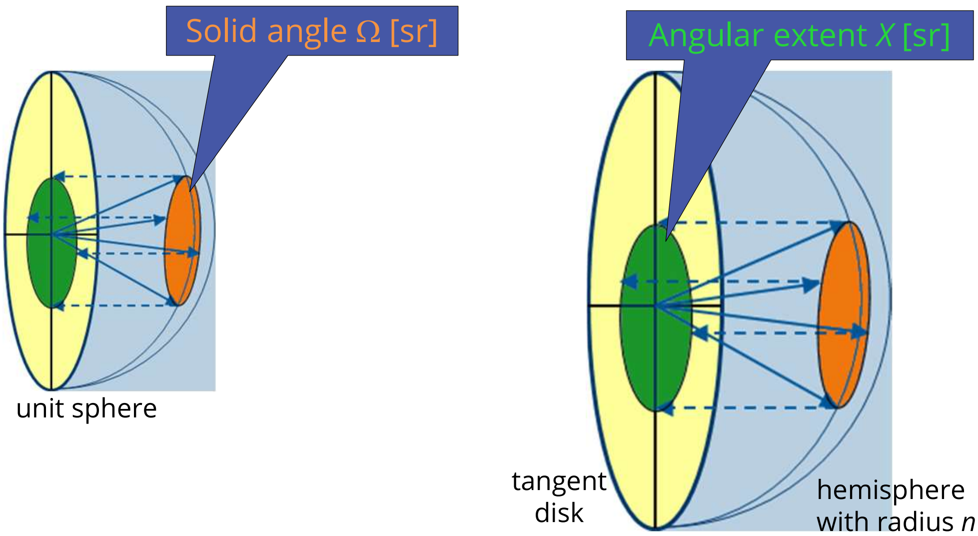
From solid angle to angular extent



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When things are simple…

Constant angular extent everywhere on planar aperture

(e.g. target for spotlight)

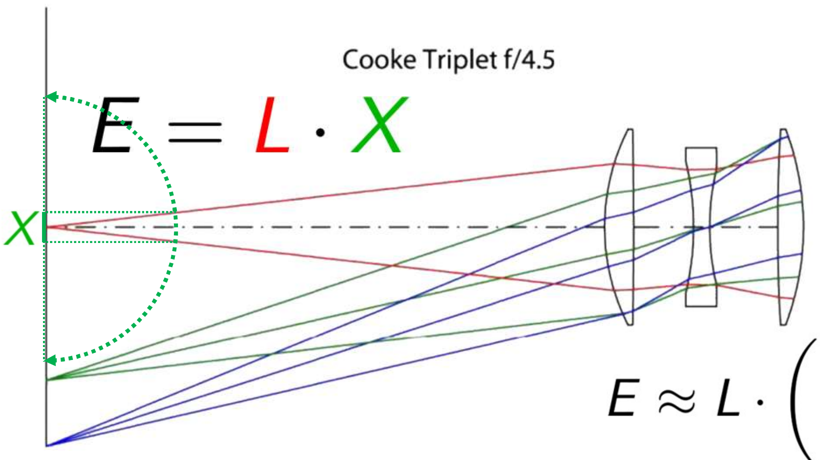
$$U = A \cdot X$$

telecentric cone: $U = A \cdot (n \sin \alpha)^2 \pi$

odd-shaped directions: $U = A \cdot n^2 \int \cos\theta \, d\Omega$

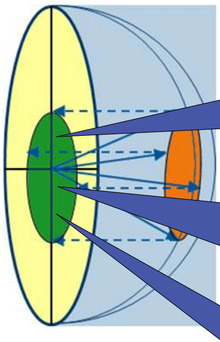
When things are simple…

Constant radiance (e.g. sensor irradiance from diffuse object)



$$E \approx L \cdot \left(\frac{1}{2F/\#}\right)^2 \pi$$

The Case for Angular extent *X*



Features prominently in etendue and radiometry relations

“**Angular extent**”
An intuitive name
Related to
but different from
“Solid angle”

“***X***”:
Otherwise unused symbol sounding like “extent”

$$\mathrm{d}X = n^2 \cos\theta \, \mathrm{d}\Omega$$

$$X = \int n^2 \cos\theta \, \mathrm{d}\Omega$$

Shortens expressions considerably

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